

Variant vector-tensor multiplets in supergravity: Classification and component reduction

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Abstract

The recent paper arXiv:1205.6881 has developed superform formulations for two versions of the vector-tensor multiplet and their Chern-Simons couplings in four-dimensional $\mathcal{N} = 2$ conformal supergravity. One of them is the standard vector-tensor multiplet with the central charge gauged by a vector multiplet. The other is the variant vector-tensor multiplet with the property that its own one-form gauges the central charge. Here a more general setup is presented in which the known versions reside as special cases. Analysis of the setup demonstrates that under certain assumptions there are two distinct variants, corresponding to the two formulations in arXiv:1205.6881. This provides a classification scheme for vector-tensor multiplets.

We then show that our superspace description leads to an efficient means of deriving component actions in supergravity. The entire action including all fermionic terms is derived for the non-linear vector-tensor multiplet. This extends the results of de Wit *et al.* in hep-th/9710212, where only the bosonic sector appeared. Finally, the bosonic sector of the action for the variant vector-tensor multiplet is given.

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1 Introduction

In $\mathcal{N} = 2$ supersymmetry there exist off-shell supermultiplets requiring the presence of a central charge, such as the Fayet-Sohnius hypermultiplet [1, 2]. The vector-tensor (VT) multiplet [3, 4] is another such supermultiplet that has received a great deal of interest due to its significance in the context of string compactifications [5].¹ Its physical fields

¹From the point of view of $\mathcal{N} = 1$ supersymmetry the multiplet decomposes into a vector and a tensor multiplet [6].

consist of a real scalar, a doublet of Weyl spinors and a gauge one-form and two-form. The multiplet may be viewed as a dual realization of the $\mathcal{N} = 2$ Abelian vector multiplet, obtained by dualizing one of the two physical scalars of the vector multiplet into a gauge two-form.

Numerous papers have been devoted to the study of the VT multiplet and its Chern-Simons couplings, both in the component approach with the use of superconformal tensor calculus [7, 8, 9] and in the framework of conventional $\mathcal{N} = 2$ superspace [10, 11, 12] as well as $\mathcal{N} = 2$ harmonic superspace [13, 14, 15, 16]. In particular, de Wit *et al.* [9] provided a comprehensive analysis of the VT multiplet and its Chern-Simons couplings in supergravity, although due its complexity only the bosonic sector of the action could be computed. This demonstrated the need for a suitable superspace formulation. An important step towards such a formulation was made by [16], where a general setting for $\mathcal{N} = 2$ rigid supersymmetric theories with gauged central charge was developed using harmonic superspace. Only recently a superfield formulation for the VT multiplet in supergravity has been found [17, 18, 19].² This delay was due in part to the absence of sufficiently simple superspace descriptions for supergravity, which has appeared in recent years [22, 23].

In addition to the standard VT multiplet, it was discovered by Theis [24, 25] that in rigid supersymmetry there exists a *variant* VT multiplet.³ This multiplet is in stark contrast to both the linear [3, 4] and non-linear [7] forms of the standard VT multiplet. It is associated with a new procedure of gauging the central charge, with the multiplet itself providing the central charge gauge one-form. This is unlike the standard gauging which makes use of a vector multiplet. The variant VT multiplet has since been generalized to supergravity [19].

In rigid supersymmetry, the difference between the versions of the VT multiplet may be illustrated in $\mathcal{N} = 2$ central charge superspace [2] in which the spinor covariant derivatives obey the anti-commutation relations

$$\{D_\alpha^i, D_\beta^j\} = -2\varepsilon_{\alpha\beta}\varepsilon^{ij}\Delta, \quad \{\bar{D}_i^{\dot{\alpha}}, \bar{D}_j^{\dot{\beta}}\} = 2\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{ij}\Delta, \quad (1.1a)$$

$$\{D_\alpha^i, \bar{D}_j^{\dot{\beta}}\} = -2i\delta_j^i\partial_\alpha^{\dot{\beta}}, \quad (1.1b)$$

where Δ is the central charge. The VT multiplets correspond to different constraints on a real superfield, L . For instance, the original *linear* VT multiplet [3, 4] can be described

²Here we are concerned with off-shell multiplets, however see [20, 21] for a different approach using Free Differential Algebra.

³In [24, 25] the variant VT multiplet was referred to as the new non-linear VT multiplet.

by the constraints [11, 13]

$$D^{ij}L = 0, \quad D_\alpha^{(i}\bar{D}^{j)}L = 0, \quad (1.2)$$

where $D^{ij} := D^{\alpha(i}D_\alpha^{j)}$ and $\bar{D}_{ij} := \bar{D}_{\dot{\alpha}(i}\bar{D}_{\dot{\alpha}}^{j)}$.⁴ It is worth mentioning that these constraints may be formulated within the superform formulations of [10, 11, 12].

Variant versions of the linear VT multiplet can be constructed by looking for consistent deformations of its constraints [14, 15, 16]. There are two possible distinct deformations (up to superfield redefinitions):

$$(i) \quad D^{ij}L = -\frac{1}{L}D^iLD^jL + \frac{1}{L}\bar{D}^iL\bar{D}^jL, \quad D_\alpha^{(i}\bar{D}^{j)}L = 0, \quad (1.3a)$$

$$(ii) \quad D^{ij}L = 2\tan(2L)D^iLD^jL + \frac{2}{\cos(2L)}\bar{D}^iL\bar{D}^jL, \quad D_\alpha^{(i}\bar{D}^{j)}L = 0. \quad (1.3b)$$

The first deformation was given in [14, 15] and at the component level corresponds to the *non-linear* VT multiplet of [7]. The self-interaction for (i) may be attributed to the presence of a Chern-Simons form. Deformation (ii) was found in [15], where it was shown that its three-form field strength cannot be locally solved in terms of a two-form gauge potential. This was why (ii) was not considered in the earlier work [14].⁵ However, later it was demonstrated by Theis [24, 25] that the one-form gauge potential corresponding to (ii) can be used to gauge the central charge, and in this way an appropriate two-form gauge potential may be defined.

In light of the three different versions of the VT multiplet in rigid supersymmetry, it is natural to ask whether they may be lifted to supergravity. Remarkably it turns out that the linear VT multiplet does not generalize to conformal supergravity nor to AdS [17], with supergravity generalizations only existing for the deformed versions.⁶ In supergravity the non-linear VT multiplet together with its general couplings was originally addressed in [9] at the component level and at the superfield level in [17, 18], with [19] elucidating the origin of the constraints.⁷ Moreover in [19] the variant VT multiplet together with its couplings to vector multiplets was constructed by using a completely superform formulation.

⁴The multiplet is on-shell if $\Delta L = 0$.

⁵S. Kuzenko, *private communication*.

⁶Although there exists a linear case of the VT multiplet in [9, 17] requiring an additional vector multiplet. It should be clear from context to which multiplet we are referring to.

⁷A linear case can exist in supergravity provided it is coupled to two vector multiplets [9, 17]. However it may be viewed as a special case of the non-linear VT multiplet with Chern-Simons terms after a superfield redefinition [18].

At present there exist only two versions of the VT multiplet within $\mathcal{N} = 2$ conformal supergravity; the non-linear VT multiplet and the variant VT multiplet. However, there still remains the possibility of alternative off-shell realizations in supergravity. For example, in [26] it was shown that it is possible to use the new procedure for gauging the central charge to construct new off-shell formulations for the *massless* Fayet-Sohnius hypermultiplet. Furthermore it would be interesting if supergravity offers new versions not available in the rigid supersymmetric case. This paper endeavors to examine this issue.

After having described VT multiplets through their superfield constraints (ensuring the appropriate component structure), an action is necessary. The linear multiplet [27] has become a powerful tool for the construction of action functionals for supergravity-matter systems, especially after its reformulation within superconformal tensor calculus [28]. In particular, in [9] the linear multiplet was used to derive the bosonic sector of the action for the standard VT multiplet.

A locally supersymmetric action based on the linear multiplet was constructed in harmonic superspace in [29]. The harmonic superspace approach to $\mathcal{N} = 2$ supergravity was developed in terms of certain prepotentials [30]. However it is not known how to relate the prepotentials to superspace differential geometry given in [31, 22, 23]. This renders it impractical for our consideration, since both the constraints and the superspace Lagrangian [17, 18] for the standard VT multiplet involve covariant derivatives. Recently, the suitable superspace action was constructed in [26]. It makes use of the ectoplasm approach [32, 33], also known as the superform approach for constructing supersymmetric invariants.⁸ One of the main results of [26] was a new action principle based on a *deformed* linear multiplet, corresponding to the case where the central charge one-form potential is not annihilated by the central charge. This is precisely the case for the variant VT multiplet whose superfield Lagrangian was given in [19]. It would therefore be interesting to derive new component results entirely from superspace. This is the other main goal of this paper.

This paper is structured as follows. In section 2 we describe a general procedure to gauge a real central charge in $\mathcal{N} = 2$ conformal supergravity. Section 3 is devoted to the general setup and superform formulation for VT multiplets. The general analysis will

⁸The mathematical formalism behind the ectoplasm approach [32, 33] is a special case of the theory of integration over surfaces in supermanifolds, see for instance [34]. The idea behind the ectoplasm approach is to use a closed superform. In four-dimensions, to the best of our knowledge this idea was first suggested by Hasler [35].

lead us to a set of constraints and possibilities will be analyzed. In section 4, building on the methods of [18], we provide component actions for two types of VT multiplets. Section 5 concludes the paper. Two technical appendices are also included. Appendix A contains a summary of the conformal supergravity formulation of [23] used in this paper and appendix B contains useful component results derived from the superspace formulation of [23].

2 Gauging the central charge in $\mathcal{N} = 2$ conformal supergravity

Off-shell formulations for VT multiplets in supergravity require the presence of a one-form to gauge the central charge. It is well known that for this purpose one can use the one-form of an off-shell vector multiplet. However, this implies that the gauge connection is annihilated by the central charge. There is a more general approach [19], inspired by the construction of Theis [24, 25], which may be used to describe the variant VT multiplet. In this section we review the approach of [19]. We make use of the superspace formulation for $\mathcal{N} = 2$ conformal supergravity developed in [23] as reformulated in [18] (see appendix A).⁹

2.1 Setup

Starting with $\mathcal{N} = 2$ conformal superspace [23], we introduce a real central charge, Δ , which we require to obey the Leibniz rule and commute with all superconformal generators and covariant derivatives¹⁰

$$[\Delta, \nabla_A] = 0 . \quad (2.1)$$

To gauge the central charge we define the gauge covariant derivatives

$$\nabla_A := \nabla_A + \mathcal{V}_A \Delta , \quad \nabla := E^A \nabla_A , \quad (2.2)$$

⁹All results derived within the formulation of [23] may be extended via gauge fixing to the formulations of [31] and [22].

¹⁰The central charge may be thought of as a derivative with respect to an extra bosonic coordinate as in [26].

where gauge connection $\mathcal{V} := E^A \mathcal{V}_A$ is not assumed to be annihilated by the central charge (*i.e.* we may have $\Delta \mathcal{V}_A \neq 0$). Then the algebra of gauge covariant derivatives is

$$\begin{aligned} [\nabla_A, \nabla_B] &= [\nabla_A, \nabla_B] + \mathcal{F}_{AB} \Delta , \\ [\Delta, \nabla_A] &= \mathcal{F}_{zA} \Delta := \Delta \mathcal{V}_A \Delta , \end{aligned} \quad (2.3)$$

where $\mathcal{F} = \frac{1}{2} E^B E^A \mathcal{F}_{AB}$ is the field-strength

$$\mathcal{F} = \nabla \mathcal{V} , \quad \mathcal{F}_{AB} = 2 \nabla_{[A} \mathcal{V}_{B]} - T_{AB}{}^C \mathcal{V}_C . \quad (2.4)$$

It proves advantageous to treat Δ on the same footing as the covariant derivatives ∇_A .¹¹ To do this we introduce the notation

$$\hat{\nabla}_{\hat{A}} := (\nabla_A, \nabla_z := \Delta) \quad (2.5)$$

and write the algebra of gauge covariant derivatives (2.3) as

$$\begin{aligned} [\hat{\nabla}_{\hat{A}}, \hat{\nabla}_{\hat{B}}] &= T_{\hat{A}\hat{B}}{}^{\hat{C}} \hat{\nabla}_{\hat{C}} + \frac{1}{2} R_{\hat{A}\hat{B}}{}^{cd} M_{cd} + R_{\hat{A}\hat{B}}{}^{kl} J_{kl} \\ &\quad + i R_{\hat{A}\hat{B}}(Y) Y + R_{\hat{A}\hat{B}}(\mathbb{D}) \mathbb{D} + R_{\hat{A}\hat{B}}{}^C K_C , \end{aligned} \quad (2.6)$$

where we define

$$T_{AB}{}^z := \mathcal{F}_{AB} , \quad T_{Az}{}^B = 0 \quad (2.7)$$

and all curvatures involving a z index vanish. Finally the Bianchi identities are

$$\hat{\nabla}_{[\hat{A}} \hat{\nabla}_{\hat{B}} \hat{\nabla}_{\hat{C}}] = 0 \quad \rightarrow \quad \hat{\nabla}_{[\hat{A}} \mathcal{F}_{\hat{B}\hat{C}}] - T_{[\hat{A}\hat{B}}{}^{\hat{D}} \mathcal{F}_{\hat{D}\hat{C}}] = 0 , \quad (2.8)$$

where $\mathcal{F}_{\hat{A}\hat{B}} := (\mathcal{F}_{AB}, \mathcal{F}_{zA})$ may be interpreted as a field strength.

Local central charge transformations are realized on the gauge covariant derivatives and tensor superfields, U , as

$$\begin{aligned} \delta_\Lambda \nabla_A &= [\Lambda \Delta, \nabla_A] \quad \Leftrightarrow \quad \delta_\Lambda \mathcal{V}_A = \Lambda \Delta \mathcal{V}_A - \nabla_A \Lambda , \\ \delta_\Lambda U &= \Lambda \Delta U , \end{aligned} \quad (2.9)$$

where the vielbein and superconformal connections are all inert under central charge transformations and $\Delta \Lambda = 0$.¹² It follows that the field strength transforms covariantly

$$\delta_\Lambda \mathcal{F}_{\hat{A}\hat{B}} = \Lambda \Delta \mathcal{F}_{\hat{A}\hat{B}} . \quad (2.10)$$

¹¹See [36] for a similar treatment in supergravity.

¹²The central charge must annihilate Λ to ensure the central charge descendant ΔU transforms covariantly under central charge transformations.

We can see that the Bianchi identities on the original superspace curvatures are unaffected by the gauging procedure and they are constrained in the usual way. However, in order to describe a background multiplet with $8 + 8$ off-shell degrees of freedom we must constrain the field strength $\mathcal{F}_{\hat{A}\hat{B}}$ in some way, which we now turn to.

2.2 The large vector multiplet

Following [19] we impose constraints at mass dimension-1 on the field strength that resemble those for a standard vector multiplet [37]

$$\mathcal{F}_{\alpha\beta}^{ij} = -2\varepsilon_{\alpha\beta}\varepsilon^{ij}\bar{M} , \quad \mathcal{F}_i^{\dot{\alpha}\dot{\beta}} = 2\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{ij}M , \quad \mathcal{F}_{\alpha j}^{i\dot{\beta}} = 0 , \quad (2.11)$$

except that M is not required to be annihilated by the central charge.¹³ The superfield M must be conformally primary with dimension 1 and $U(1)$ charge -2 (a Jacobi identity implies that M is conformally primary [23])

$$K_A M = 0 , \quad \mathbb{D}M = M , \quad YM = -2M . \quad (2.12)$$

The Bianchi identities on the field strength then lead to the constraints

$$\nabla_\alpha^{(i}\bar{\nabla}_{\dot{\alpha}}^{j)} \ln \left(\frac{M}{\bar{M}} \right) = 0 , \quad (2.13)$$

$$\bar{M}\nabla^{ij} \left(\frac{M}{\bar{M}} \right) = M\bar{\nabla}^{ij} \left(\frac{\bar{M}}{M} \right) , \quad (2.14)$$

where

$$\nabla^{ij} := \nabla^{\alpha(i}\nabla_{\alpha}^{j)} , \quad \bar{\nabla}^{ij} := \bar{\nabla}_{\dot{\alpha}}^{(i}\bar{\nabla}^{j)\dot{\alpha}} . \quad (2.15)$$

The remaining components of \mathcal{F} are

$$\mathcal{F}_{z\alpha}^i = \nabla_\alpha^i \ln \bar{M} , \quad (2.16a)$$

$$\mathcal{F}_{a\beta}^j = \frac{i}{2}(\sigma_a)_{\beta}^{\dot{\alpha}} M \bar{\nabla}_{\dot{\alpha}}^j \left(\frac{\bar{M}}{M} \right) , \quad (2.16b)$$

$$\mathcal{F}_{za} = -\frac{i}{8}(\sigma_a)_{\alpha\dot{\alpha}} (\nabla^{\alpha k} \bar{\nabla}_{\dot{\alpha}}^k \ln M + \bar{\nabla}_{\dot{\alpha}}^k \nabla^{\alpha k} \ln \bar{M}) , \quad (2.16c)$$

$$\mathcal{F}_{ab} = -\frac{1}{8}(\sigma_{ab})^{\alpha\beta} (\bar{M}\nabla_{\alpha\beta} \left(\frac{M}{\bar{M}} \right) + 4\bar{M}W_{\alpha\beta}) + \text{c.c.} \quad (2.16d)$$

It is interesting to note that the expression for $\mathcal{F}_{z\alpha}^i$ implies that the operator $\bar{M}\Delta$ commutes with ∇_α^i

$$[\bar{M}\Delta, \nabla_\alpha^i] = 0 , \quad [M\Delta, \bar{\nabla}_i^{\dot{\alpha}}] = 0 . \quad (2.17)$$

¹³The definition of M differs by a factor of i from [19] and by a minus sign from [26].

The resulting multiplet is called the large vector multiplet, due to the fact that it contains too many component fields to correspond to a multiplet of $8 + 8$ degrees of freedom. Therefore it is necessary to constrain the component fields of M in addition to the constraints (2.13). There are two known choices of significance.

One obvious choice is to let M be independent of the central charge, which leads us to $M = \mathcal{Z}$ with \mathcal{Z} a vector multiplet

$$\bar{\nabla}_{\dot{\alpha}}^i \mathcal{Z} = 0 \ , \quad \nabla^{ij} \mathcal{Z} = \bar{\nabla}^{ij} \bar{\mathcal{Z}} \ . \quad (2.18)$$

Then the field strength becomes $\mathcal{F} = F$, the field strength of a vector multiplet¹⁴

$$F_{a\beta}^j = \frac{i}{2}(\sigma_a)_{\beta}^{\dot{\gamma}} \bar{\nabla}_{\dot{\gamma}}^j \bar{\mathcal{Z}} \ , \quad F_{aj}^{\dot{\beta}} = -\frac{i}{2}(\sigma_a)_{\gamma}^{\dot{\beta}} \nabla_j^{\gamma} \mathcal{Z} \ , \quad (2.19a)$$

$$F_{ab} = -\frac{1}{8}(\sigma_{ab})_{\alpha\beta}(\nabla^{\alpha\beta} \mathcal{Z} + 4W^{\alpha\beta} \bar{\mathcal{Z}}) + \frac{1}{8}(\tilde{\sigma}_{ab})_{\dot{\alpha}\dot{\beta}}(\bar{\nabla}^{\dot{\alpha}\dot{\beta}} \bar{\mathcal{Z}} + 4\bar{W}^{\dot{\alpha}\dot{\beta}} \mathcal{Z}) \ . \quad (2.19b)$$

This is the standard procedure for gauging the central charge.

Another choice for the superfield is

$$M = i\mathcal{Z}e^{-iL} \ , \quad (2.20)$$

where L is a real primary superfield not annihilated by the central charge and with zero dilatation and $U(1)$ weight. In this case the components of \mathcal{F} were found in [19] to be

$$\begin{aligned} \mathcal{F}_{z\alpha}^i &= i\nabla_{\alpha}^i L \ , \quad \mathcal{F}_{zi}^{\dot{\alpha}} = -i\bar{\nabla}_{\dot{i}}^{\dot{\alpha}} L \ , \\ \mathcal{F}_{a\beta}^j &= \frac{1}{2}e^{-iL}(\sigma_a)_{\beta}^{\dot{\alpha}} \bar{\nabla}_{\dot{\alpha}}^j (\bar{\mathcal{Z}}e^{2iL}) \ , \quad \mathcal{F}_{aj}^{\dot{\beta}} = \frac{1}{2}e^{iL}(\sigma_a)_{\alpha}^{\dot{\beta}} \nabla_j^{\alpha} (\mathcal{Z}e^{-2iL}) \ , \\ \mathcal{F}_{az} &= \frac{1}{8}(\sigma_a)_{\gamma\dot{\gamma}}[\nabla^{\gamma k}, \bar{\nabla}_{\dot{k}}^{\dot{\gamma}}]L \ , \\ \mathcal{F}_{ab} &= -\frac{i}{8}(\sigma_{ab})^{\alpha\beta}e^{iL}(\nabla_{\alpha\beta}(\mathcal{Z}e^{-2iL}) - 4W_{\alpha\beta}\bar{\mathcal{Z}}) + \text{c.c.} \ , \end{aligned} \quad (2.21)$$

where L is constrained by

$$\nabla_{\alpha}^{(i} \bar{\nabla}_{\dot{\alpha}}^{j)} L = 0 \ , \quad e^{iL} \nabla^{ij} (\mathcal{Z}e^{-2iL}) + e^{-iL} \bar{\nabla}^{ij} (\bar{\mathcal{Z}}e^{2iL}) = 0 \ . \quad (2.22)$$

Introducing an additional constraint, the simplest version of which is

$$e^{-iL} \nabla^{ij} (\mathcal{Z}e^{2iL}) + e^{iL} \bar{\nabla}^{ij} (\bar{\mathcal{Z}}e^{-2iL}) = 0 \quad (2.23)$$

guarantees the existence of a gauge two-form and ensures that L describes a multiplet with $8 + 8$ degrees of freedom, known as the variant vector-tensor multiplet [19]. In the rigid supersymmetric limit it reduces to the multiplet constructed by Theis [24, 25].

¹⁴Throughout this paper, additional abelian vector multiplets, \mathcal{W} , are described by a similar two-form field strength with the same constraints except with \mathcal{Z} replaced with \mathcal{W} .

3 Superform formulation for vector-tensor multiplets

As we saw in the previous section, allowing the central charge gauge field to have non-trivial action under the central charge lead us to new possibilities that are not fixed by the Bianchi identities. We may fix these extra degrees of freedom by choosing additional constraints. In light of the role that the central charge multiplet plays for the VT multiplet, we now turn to fixing these degrees of freedom by demanding the existence of a multiplet with a one-form and a two-form gauge potential.

In this section we will make use of a superform formulation and examine the possibilities allowed by the constraints. The formulation will be a generalization of the one in [19], where superform formulations for both known versions of the VT multiplet in conformal supergravity was given. Our approach has a number of advantages. In particular, it makes manifest the existence of two gauge fields (the one-form and the two-form), without the need to go to components and it allows the constraints on superfields to be derived from the superform Bianchi identities. Superform formulations are also very useful tools for component reduction (see section 4), where the closure of supersymmetry transformations of the component fields are guaranteed by the Bianchi identities.

3.1 One-form geometry

In addition to the central charge gauge one-form we introduce a new gauge one-form, $\tilde{\mathcal{V}} := E^A \tilde{\mathcal{V}}_A$, which is also not required to be annihilated by the central charge. Its transformation law is defined as (compare with section 2)

$$\delta \tilde{\mathcal{V}} = \Lambda \Delta \tilde{\mathcal{V}} + d\tilde{\Gamma} , \quad \Delta \tilde{\Gamma} = 0 , \quad (3.1)$$

with $\tilde{\Gamma}$ the gauge parameter associated with $\tilde{\mathcal{V}}$. The corresponding field strength $\tilde{\mathcal{F}} = \frac{1}{2} E^B E^A \tilde{\mathcal{F}}_{AB}$ is given by

$$\tilde{\mathcal{F}} = \nabla \tilde{\mathcal{V}} , \quad \tilde{\mathcal{F}}_{AB} = 2 \nabla_{[A} \tilde{\mathcal{V}}_{B]} - T_{AB}{}^C \tilde{\mathcal{V}}_C . \quad (3.2)$$

and transforms homogeneously

$$\delta \tilde{\mathcal{F}} = \Lambda \Delta \tilde{\mathcal{F}} . \quad (3.3)$$

As in [19] we may extend the field strength to $\tilde{\mathcal{F}}_{\hat{A}\hat{B}} = (\tilde{\mathcal{F}}_{AB}, \tilde{\mathcal{F}}_{zA} := \Delta \tilde{\mathcal{V}}_A)$ and verify that the Bianchi identities may be written as

$$\nabla_{[\hat{A}} \tilde{\mathcal{F}}_{\hat{B}\hat{C}]} - T_{[\hat{A}\hat{B}}{}^{\hat{D}} \tilde{\mathcal{F}}_{|\hat{D}|\hat{C}]} = 0 . \quad (3.4)$$

We now impose the constraints on the field strength

$$\tilde{\mathcal{F}}_{\alpha\beta}^{ij} = -2\varepsilon_{\alpha\beta}\varepsilon^{ij}\bar{N} , \quad \tilde{\mathcal{F}}_i^{\dot{\alpha}\dot{\beta}} = 2\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{ij}N , \quad \tilde{\mathcal{F}}_{\alpha j}^{i\dot{\beta}} = 0 , \quad (3.5)$$

where N is a primary superfield with dimension 1 and $U(1)$ charge -2 . The Bianchi identities (3.4) may be solved giving the field strength components

$$\begin{aligned} \tilde{\mathcal{F}}_{z\alpha}^i &= \frac{1}{M}\nabla_\alpha^i\bar{N} , \\ \tilde{\mathcal{F}}_{a\beta}^j &= -\frac{i}{2}(\sigma_a)_{\beta\dot{\alpha}}(\bar{\nabla}^{\dot{\alpha}j}\bar{N} - \frac{\bar{M}}{M}\bar{\nabla}^{\dot{\alpha}j}N) , \\ \tilde{\mathcal{F}}_{az} &= \frac{i}{8}(\sigma_a)_{\gamma\dot{\gamma}}\left(\nabla^{\gamma k}\left(\frac{1}{M}\bar{\nabla}_k^{\dot{\gamma}}N\right) + \bar{\nabla}_k^{\dot{\gamma}}\left(\frac{1}{M}\nabla^{\gamma k}\bar{N}\right)\right) , \\ \tilde{\mathcal{F}}_{ab} &= -\frac{1}{8}(\sigma_{ab})^{\alpha\beta}\left(\nabla_{\alpha\beta}N - 2\nabla_\alpha^k\left(\frac{M}{\bar{M}}\right)\nabla_{\beta k}\bar{N} - \frac{M}{\bar{M}}\nabla_{\alpha\beta}\bar{N} + 4\bar{N}W_{\alpha\beta}\right) + \text{c.c.} \end{aligned} \quad (3.6)$$

and the superfield constraints

$$\nabla_\alpha^{(i}\left(\frac{1}{M}\bar{\nabla}_{\dot{\beta}}^{j)}N\right) + \bar{\nabla}_{\dot{\beta}}^{(i}\left(\frac{1}{M}\nabla_\alpha^{j)}\bar{N}\right) = 0 , \quad (3.7a)$$

$$\bar{N}\nabla^{ij}\left(\frac{M}{\bar{M}}\right) + \nabla^{ij}N - \nabla^{ij}\left(\frac{\bar{N}M}{\bar{M}}\right) = N\bar{\nabla}^{ij}\left(\frac{\bar{M}}{M}\right) + \bar{\nabla}^{ij}\bar{N} - \bar{\nabla}^{ij}\left(\frac{N\bar{M}}{M}\right) . \quad (3.7b)$$

These results represent new general couplings of the large vector multiplet to a one-form and generalizes the one-form geometry in [19] for both versions of the VT multiplet.

3.2 Two-form geometry

The above system of coupled one-forms contain too many component fields. In order to constrain the component fields we demand the existence of a two-form, and hence provide a general framework for both versions of the VT multiplet. Similar to the analysis in [19], we introduce a gauge two-form, $B = \frac{1}{2}E^B E^A B_{AB}$, with corresponding field strength

$$H := \nabla B - \frac{1}{2}\eta_{IJ}\tilde{\mathcal{V}}^I\nabla\tilde{\mathcal{V}}^J , \quad (3.8)$$

where we couple a number of one-forms, $\tilde{\mathcal{V}}^I$, to the two-form via the coupling constants η_{IJ} .¹⁵ The transformation law of B is defined to be

$$\delta B = \Lambda\Delta B + \frac{1}{2}\eta_{IJ}\tilde{\Gamma}^I d\tilde{\mathcal{V}}^J + d\Xi , \quad \Delta\Xi = 0 , \quad (3.9)$$

¹⁵Here η_{IJ} is not required to be symmetric since it was not chosen to be symmetric in [9].

where the one-form Ξ is the gauge parameter for B . As in [19] we may extend the field strength to $H_{\hat{A}\hat{B}\hat{C}} = (H_{ABC}, H_{zAB} := \Delta B_{AB} + \eta_{IJ} \tilde{\mathcal{V}}_{[A}^I \tilde{\mathcal{F}}_{|z|B]}^J)$, which transforms homogeneously

$$\delta H_{\hat{A}\hat{B}\hat{C}} = \Lambda \Delta H_{\hat{A}\hat{B}\hat{C}} \quad (3.10)$$

and satisfies the Bianchi identity

$$\nabla_{[\hat{A}} H_{\hat{B}\hat{C}\hat{D}]} - \frac{3}{2} T_{[\hat{A}\hat{B}} \varepsilon H_{|\varepsilon|\hat{C}\hat{D}]} + \frac{3}{4} \eta_{IJ} \tilde{\mathcal{F}}_{[\hat{A}\hat{B}}^I \tilde{\mathcal{F}}_{\hat{C}\hat{D}]}^J = 0 . \quad (3.11)$$

We now impose constraints on the field strength similar to those in [19]

$$\begin{aligned} H_{\alpha\beta\gamma}^{ijk} &= H_{i\ j\ k}^{\dot{\alpha}\dot{\beta}\dot{\gamma}} = H_{\alpha\beta k}^{i\ j\ \dot{\gamma}} = H_{\alpha j\ k}^{i\ \dot{\beta}\dot{\gamma}} = 0 , \\ H_{a\beta\gamma}^{jk} &= H_{a\ j\ k}^{\dot{\beta}\dot{\gamma}} = 0 , \quad H_{a\beta k}^{j\ \dot{\gamma}} = -2i\delta_k^j (\sigma_a)_\beta{}^{\dot{\gamma}} \tilde{H} , \end{aligned} \quad (3.12)$$

where \tilde{H} is some real superfield. Without choosing the form of M or N we may analyze (3.11) to find the components of $H_{\hat{A}\hat{B}\hat{C}}$:

$$\begin{aligned} H_{a\beta\gamma} &= H_{a\dot{\beta}\dot{\gamma}} = 0 , \quad H_{a\beta k}^{j\ \dot{\gamma}} = -2i\delta_k^j (\sigma_a)_\beta{}^{\dot{\gamma}} \tilde{H} , \\ H_{z\alpha\beta}^{ij} &= -\varepsilon^{ij} \varepsilon_{\alpha\beta} \eta_{IJ} \frac{\bar{N}^I \bar{N}^J}{\bar{M}} , \quad H_{zi\ j}^{\dot{\alpha}\dot{\beta}} = \varepsilon_{ij} \varepsilon^{\dot{\alpha}\dot{\beta}} \eta_{IJ} \frac{N^I N^J}{M} , \quad H_{z\alpha j}^{i\ \dot{\beta}} = 0 , \\ H_{az\beta}^j &= -\frac{i}{4} \eta_{IJ} (\sigma_a)_{\beta\dot{\alpha}} \left(\frac{2}{M} \bar{\nabla}^{\dot{\alpha}j} N^{(I} \bar{N}^{J)} - \frac{1}{M\bar{M}} \bar{\nabla}^{\dot{\alpha}j} M \bar{N}^I \bar{N}^J - \bar{\nabla}^{\dot{\alpha}j} \left(\frac{\bar{N}^I \bar{N}^J}{M} \right) \right) , \\ H_{ab\gamma}^k &= 2(\sigma_{ab})_\gamma{}^\alpha \nabla_\alpha^k \tilde{H} , \quad H_{abk}^{\dot{\gamma}} = 2(\tilde{\sigma}_{ab})^{\dot{\gamma}}{}_{\dot{\alpha}} \bar{\nabla}_{\dot{\alpha}}^k \tilde{H} , \\ H_{abz} &= -\frac{1}{16} \eta_{IJ} (\sigma_{ab})^{\alpha\beta} \left(\nabla_{\alpha\beta} \left(\frac{N^I N^J}{M} \right) + 4W_{\alpha\beta} \frac{\bar{N}^I \bar{N}^J}{\bar{M}} - \frac{2}{M} \nabla_{\alpha\beta} \bar{N}^{(I} N^{J)} \right. \\ &\quad \left. + \frac{1}{M\bar{M}} \nabla_{\alpha\beta} \bar{M} N^I N^J - \frac{4}{M} \nabla_\alpha^k N^{(I} \nabla_{\beta k} \bar{N}^{J)} + \frac{2}{M} \nabla_\alpha^k \bar{M} \nabla_{\beta k} \left(\frac{N^I N^J}{M} \right) \right. \\ &\quad \left. + \frac{2M}{\bar{M}^2} \nabla_\alpha^k \bar{N}^I \nabla_{\beta k} \bar{N}^J \right) + \text{c.c.} , \\ H_{abc} &= -\frac{1}{32} \varepsilon_{abcd} (\sigma^d)^{\alpha\dot{\alpha}} \left(2[\nabla_\alpha^k, \bar{\nabla}_{\dot{\alpha}k}] \tilde{H} \right. \\ &\quad \left. + \eta_{IJ} M \bar{\nabla}_{\dot{\alpha}}^k \left(\frac{\bar{M}}{M} \right) \left(\frac{2}{M} \nabla_{\alpha k} \bar{N}^I N^J - \frac{1}{M\bar{M}} \nabla_{\alpha k} \bar{M} N^I N^J - \nabla_{\alpha k} \left(\frac{N^I N^J}{M} \right) \right) \right. \\ &\quad \left. + \eta_{IJ} (\bar{\nabla}_{\dot{\alpha}}^k \bar{N}^I - \frac{\bar{M}}{M} \bar{\nabla}_{\dot{\alpha}}^k N^I) \left(\nabla_{\alpha k} N^J - \frac{M}{\bar{M}} \nabla_{\alpha k} \bar{N}^J \right) \right) + \text{c.c.} \end{aligned} \quad (3.13)$$

In addition to the constraints imposed from the one-form geometry we also find the

constraints:

$$\tilde{H} = -\frac{1}{2}\eta_{IJ}M\bar{M}Y^IY^J, \quad Y^I := \frac{1}{2i}\left(\frac{N^I}{M} - \frac{\bar{N}^I}{\bar{M}}\right), \quad (3.14a)$$

$$0 = \eta_{IJ}G^{IJij},$$

$$\begin{aligned} -4G^{IJij} = & \frac{i}{2}\nabla^{ij}\left(\frac{N^IN^J}{M}\right) + \frac{i}{2M\bar{M}}\nabla^{ij}\bar{M}N^IN^J - \frac{i}{\bar{M}}\nabla^{ij}\bar{N}^{(I}N^{J)} \\ & - \frac{2i}{\bar{M}}\nabla^{\gamma(i}N^{(I}\nabla_{\gamma}^{j)}\bar{N}^{J)} + \frac{i}{\bar{M}}\nabla^{\gamma(i}\bar{M}\nabla_{\gamma}^{j)}\left(\frac{N^IN^J}{M}\right) \\ & + \frac{iM}{M^2}\nabla^{\gamma(i}\bar{N}^I\nabla_{\gamma}^{j)}\bar{N}^J + \text{c.c.}, \end{aligned} \quad (3.14b)$$

$$0 = \eta_{IJ}[\bar{\nabla}^{\dot{\alpha}(k}M\nabla^{\alpha l)}\bar{N}^{(I} + \nabla^{\alpha(k}\bar{M}\bar{\nabla}^{\dot{\alpha}l)}N^{(I]}Y^{J)}]. \quad (3.14c)$$

The last constraint (3.14c) is quite restrictive when it comes to the choice of M and N . It holds if

$$Y^I = \frac{1}{2i}\left(\frac{N^I}{M} - \frac{\bar{N}^I}{\bar{M}}\right) = 0 \quad (3.15)$$

or

$$\bar{\nabla}^{\dot{\alpha}(k}M\nabla^{\alpha l)}\bar{N}^I + \nabla^{\alpha(k}\bar{M}\bar{\nabla}^{\dot{\alpha}l)}N^I = 0. \quad (3.16)$$

However the case where $Y^I = 0$ is degenerate since N^I drops out of all components of the two-form field strength H_{ABC} and is unconstrained by the two-form geometry. The other possibility (3.16) is satisfied if either M or N^I are chiral or if N^I/M is a real constant. However if N^I/M is a real constant then $Y^I = 0$. Thus we are left with two cases to consider, one where M is chiral and the other where N is chiral.

If M is chiral the large vector multiplet reduces to the usual vector multiplet, $M = \mathcal{Z}$. We may then represent any of the N^I as¹⁶

$$N = \mathcal{W}(R + i\tilde{L}), \quad (3.17)$$

with R and \tilde{L} real superfields and \mathcal{W} a vector multiplet. It can be seen that the superfield R drops out of the H_{ABC} components and out of the constraints (3.7a), (3.7b) and (3.14b).¹⁷ Here we wish to describe multiplets with 8+8 degrees of freedom and therefore we will choose to freeze R to a constant. After freezing R , in terms of the superfield constraints there are two distinct choices for N

$$(i) \ N = i\mathcal{W}\tilde{L}, \quad (ii) \ N = \mathcal{W} \quad (3.18)$$

¹⁶Here we suppress the index I to avoid awkward notation.

¹⁷Although the one-form and its field strength has non-trivial dependence on R .

up to shifts in \tilde{L} and \mathcal{W} . However, the choice (i) leads to the same form of constraints on \tilde{L} as the choice

$$N = i\mathcal{Z}L, \quad L = \left(\frac{\mathcal{W}}{\mathcal{Z}} + \frac{\bar{\mathcal{W}}}{\bar{\mathcal{Z}}}\right)\tilde{L}. \quad (3.19)$$

Therefore we may set $N^I = (i\mathcal{Z}L, \mathcal{W}^{\hat{I}})$ and doing so recovers the constraints for the VT multiplet of [9]

$$\nabla_{\alpha}^{(i} \bar{\nabla}_{\dot{\beta}}^{j)} L = 0, \quad (3.20a)$$

$$\nabla^{ij}(\mathcal{Z}L) + \bar{\nabla}^{ij}(\bar{\mathcal{Z}}L) - L\nabla^{ij}\mathcal{Z} = 0, \quad (3.20b)$$

$$\eta_{IJ}G^{IJij} = 0, \quad (3.20c)$$

with

$$G^{IJij} = \frac{1}{2}[i\mathcal{Z}\nabla^{ij}Y^{(I}Y^{J)} + 2i\mathcal{Z}\nabla^{\gamma(i}Y^{(I}\nabla_{\gamma}^{j)}Y^{J)} + 2iY^{(I}\nabla^{\gamma(i}Y^{J)}\nabla_{\gamma}^{j)}\mathcal{Z}] + \text{c.c.}, \quad (3.21)$$

which appeared in [17, 18].¹⁸

On the other hand, if N^I are chiral then the relations (3.7) constrain them to be reduced chiral. Now we represent M as

$$M = i\mathcal{Z}Re^{-iL}, \quad (3.22)$$

with R and L real superfields. Similar to the previous case, R formally drops out of the H_{ABC} components and out of the constraints (3.7a), (3.7b) and (3.14b). This time we simply make the choice $R = 1$. Then letting $N^I = \mathcal{Z}, \mathcal{W}^{\hat{I}}$ recovers the variant VT multiplet of [19]:

$$\nabla_{\alpha}^{(i} \bar{\nabla}_{\dot{\beta}}^{j)} L = 0, \quad (3.23a)$$

$$e^{iL}\nabla^{ij}(\mathcal{Z}e^{-2iL}) + e^{-iL}\bar{\nabla}^{ij}(\bar{\mathcal{Z}}e^{2iL}) = 0, \quad (3.23b)$$

$$e^{-iL}\nabla^{ij}(\mathcal{Z}Ke^{2iL}) + e^{iL}\bar{\nabla}^{ij}(\bar{\mathcal{Z}}\bar{K}e^{-2iL}) = 0, \quad K \equiv \eta_{11} + \eta_{1\hat{I}}\frac{\mathcal{W}^{\hat{I}}}{\mathcal{Z}} + \eta_{\hat{I}\hat{J}}\frac{\mathcal{W}^{\hat{I}}\mathcal{W}^{\hat{J}}}{\mathcal{Z}^2}. \quad (3.23c)$$

A main point of our analysis is that one cannot have both kinds of VT multiplets coupled to each other. As a result the two cases from here on will be classified as the type I and the type II VT multiplets respectively. Their corresponding superfield Lagrangians are given in [17, 18, 19].

¹⁸Here we adopt the convention $\eta_{\hat{I}\hat{1}} = 0$ in [9], although it may be more natural to define η_{IJ} to be symmetric.

4 Vector-tensor multiplet in components

In the past $\mathcal{N} = 2$ superconformal tensor calculus [38, 39, 40] has been very popular in constructing component actions for matter multiplets in supergravity, *e.g.* [41, 42, 9]. However as highlighted by the complexity of the results in [9], a superfield formulation is desired after due its simpler form. Moreover, it proves advantageous to have superform formulations for the supermultiplets. Besides being simpler, superform formulations are useful in identifying covariant field strengths and have an important role to play for the ectoplasm (or superform) approach [32, 33]. In [18] the superspace formulations for supergravity of [22, 23] were used to efficiently construct component actions entirely from superspace, with superforms for the vector and tensor multiplets facilitating the component reduction.¹⁹ In this section we will make use of the techniques of [18] and the action principle in [26] to derive new component results for VT multiplets.

4.1 Weyl multiplet

The Weyl multiplet [38, 39, 40] may be described by the superspace formulation of [23]. It contains a set of one-forms: the vierbein e_m^a , the gravitino $\psi_{m_i}^\alpha$, the $U(1)_R$ and $SU(2)_R$ gauge fields A_m and ϕ_m^{ij} and the dilatation gauge field b_m . These are simply the component projections of corresponding superforms

$$\begin{aligned} e_m^a &:= E_m^a|, & \psi_{m_i}^\alpha &:= 2E_{m_i}^\alpha|, & \bar{\psi}_{m\dot{\alpha}}^i &:= 2E_{m\dot{\alpha}}^i|, \\ A_m &:= \Phi_m|, & \phi_m^{ij} &:= \Phi_m^{ij}|, & b_m &:= B_m|, \end{aligned} \quad (4.1)$$

where we define the component projection of a superfield, $V(z)$ by

$$V(z)| := V(z)|_{\theta_i=\bar{\theta}^i=0}. \quad (4.2)$$

The component gauge fields for Lorentz, special conformal and S -supersymmetry transformations are denoted by ω_m^{ab} , $\phi_{m\alpha}^i$ and \mathfrak{f}_m^a respectively:

$$\omega_m^{ab} := \Omega_m^{ab}|, \quad \phi_{m\alpha}^i := 2\mathfrak{F}_{m\alpha}^i|, \quad \bar{\phi}_{m\dot{\alpha}}^i := 2\mathfrak{F}_{m\dot{\alpha}}^i|, \quad \mathfrak{f}_m^a := \mathfrak{F}_m^a|. \quad (4.3)$$

The constraints on the superspace curvatures imply that they are composite objects (see appendix B). Finally, the torsion component $W_{\alpha\beta}$ provides additional non-gauge compo-

¹⁹The superspace formulation of [22] may be derived by a gauge fixing of [23].

nent fields

$$W_{ab}^+ := (\sigma_{ab})^{\alpha\beta} W_{\alpha\beta} | , \quad W_{ab}^- := -(\tilde{\sigma}_{ab})_{\dot{\alpha}\dot{\beta}} \bar{W}^{\dot{\alpha}\dot{\beta}} | , \quad (4.4a)$$

$$\Sigma^{\alpha i} := \frac{1}{3} \nabla_{\beta}^i W^{\alpha\beta} | , \quad \bar{\Sigma}_{\dot{\alpha} i} := -\frac{1}{3} \bar{\nabla}_{\dot{\beta} i} \bar{W}^{\dot{\alpha}\dot{\beta}} | , \quad (4.4b)$$

$$D := \frac{1}{12} \nabla^{\alpha\beta} W_{\alpha\beta} | = \frac{1}{12} \bar{\nabla}_{\dot{\alpha}\dot{\beta}} \bar{W}^{\dot{\alpha}\dot{\beta}} | . \quad (4.4c)$$

In what follows (as in [18]) we may drop the component projection operator on $W_{\alpha\beta}$ when it is clear from the context that we are referring to its component projection.

4.2 Deformed linear multiplet and action principle

The linear multiplet [2] is an off-shell representation of $\mathcal{N} = 2$ supersymmetry with central charge. In supergravity it was used to construct the component action for the type I VT multiplet [9]. However a modification is needed when the central charge is gauged in a non-standard way (such as for the type II VT multiplet). It requires a deformed linear multiplet [26], which generalizes the usual linear multiplet in the presence of a background large vector multiplet.

The deformed linear multiplet is described by a real symmetric superfield, $\mathcal{L}^{ji} = \mathcal{L}^{ij} = (\mathcal{L}_{ij})^*$ constrained by

$$\tilde{\nabla}_{\alpha}^{(i} \mathcal{L}^{jk)} = 0 , \quad \tilde{\nabla}_{(\dot{i}}^{\dot{\alpha}} \mathcal{L}_{jk)} = 0 , \quad (4.5)$$

where the tilded derivatives are defined as

$$\tilde{\nabla}_{\alpha}^i = \bar{M}^{-1} \nabla_{\alpha}^i \bar{M} , \quad \tilde{\nabla}_{\dot{i}}^{\dot{\alpha}} = M^{-1} \bar{\nabla}_{\dot{i}}^{\dot{\alpha}} M . \quad (4.6)$$

Its component fields are given by

$$\ell^{ij} := \mathcal{L}^{ij} | , \quad (4.7a)$$

$$\chi_{\alpha i} := \frac{1}{3} \tilde{\nabla}_{\alpha}^j \mathcal{L}_{ij} | , \quad \bar{\chi}^{\dot{\alpha} i} := \frac{1}{3} \tilde{\nabla}_{\dot{j}}^{\dot{\alpha}} \mathcal{L}^{ij} | , \quad (4.7b)$$

$$F := \frac{1}{12} \tilde{\nabla}^{ij} \mathcal{L}_{ij} | , \quad \bar{F} := \frac{1}{12} \tilde{\nabla}^{ij} \mathcal{L}_{ij} | , \quad (4.7c)$$

with an additional component (a three-form) associated with the superform $\hat{\Sigma}_{5ABC}$ in [26]

$$\hat{\Sigma}_{5\alpha\beta\gamma}^{ijk} = \hat{\Sigma}_{5\alpha\beta k}^{ij\dot{\gamma}} = \hat{\Sigma}_{5a\beta\gamma}^{ij} = 0 , \quad \hat{\Sigma}_{5a\beta k}^{j\dot{\gamma}} = 2(\sigma_a)_{\beta}^{\dot{\gamma}} \mathcal{L}^j_k , \quad (4.8a)$$

$$\hat{\Sigma}_{5ab\alpha}^i = \frac{2i}{3} (\sigma_{ab})_{\alpha}^{\gamma} \tilde{\nabla}_{\gamma}^k \mathcal{L}_k^i , \quad (4.8b)$$

$$\hat{\Sigma}_{5abc} = \frac{i}{24} \varepsilon_{abcd} (\tilde{\sigma}^d)^{\dot{\alpha}\alpha} [\tilde{\nabla}_{\alpha}^k, \tilde{\nabla}_{\dot{\alpha}}^l] \mathcal{L}_{kl} . \quad (4.8c)$$

Coupling $\hat{\Sigma}_{5ABC}$ to a four-form $\hat{\Sigma}_{ABCD}$ and using the ectoplasm approach leads to an action principle for the deformed linear multiplet [26]

$$S = \int d^4x e \left(F\phi + \chi_i^\alpha \lambda_\alpha^i + \frac{1}{8} \ell^{ij} X_{ij} - v_a \tilde{\Sigma}^a - \frac{i}{2} \psi_{\alpha\dot{\alpha}i} (2\bar{\chi}^{\dot{\alpha}i} \bar{\phi} + \ell^{ij} \bar{\lambda}_{\dot{\alpha}j}) + (\sigma^{cd})_{\gamma\delta} \psi_{c\dot{k}}^\gamma \psi_{d\dot{l}}^\delta \ell^{kl} \bar{\phi} + \text{c.c.} \right), \quad (4.9)$$

where

$$\tilde{\Sigma}^a = \frac{1}{6} e_m{}^a \varepsilon^{mnpq} \hat{\Sigma}_{5npq} | \quad (4.10)$$

and

$$\phi := M |, \quad \lambda_\alpha^i := \bar{M} \nabla_\alpha^i \left(\frac{M}{\bar{M}} \right) |, \quad X^{ij} := \bar{M} \nabla^{ij} \left(\frac{M}{\bar{M}} \right) | = (X_{ij})^*, \quad v_a = e_a{}^m \mathcal{V}_m | \quad (4.11)$$

are component fields of the large vector multiplet.²⁰ The action has the same form as that for a standard linear multiplet [43], however the supersymmetry transformations of the component fields have been modified.

On a final note we may construct $\tilde{\Sigma}^a$ from the superform $\hat{\Sigma}_{5ABC}$. Making use of the superspace identity

$$\hat{\Sigma}_{5mnp} = E_m{}^A E_n{}^B E_p{}^C \hat{\Sigma}_{5ABC} (-)^{ab+ac+bc} \quad (4.12)$$

and taking its superspace projection we find

$$\tilde{\Sigma}^a = \mathfrak{E}^a + (\sigma^{ab})_\alpha{}^\beta \psi_{b\dot{k}}^\alpha \chi_\beta^k - (\tilde{\sigma}^{ab})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{\psi}_{b\dot{\alpha}}^k \bar{\chi}_{\dot{\beta}}^{\dot{k}} - \frac{1}{2} \varepsilon^{abcd} (\sigma_b)_{\alpha}{}^{\dot{\alpha}} \psi_{c\dot{k}}^\alpha \bar{\psi}_{d\dot{\alpha}}^l \ell^k{}_l, \quad (4.13)$$

with

$$\mathfrak{E}^a := \frac{1}{6} \varepsilon^{abcd} \hat{\Sigma}_{5bcd} | = \frac{i}{24} (\tilde{\sigma}^a)^{\dot{\alpha}\alpha} [\tilde{\nabla}_\alpha^k, \tilde{\nabla}_{\dot{\alpha}}^l] \mathcal{L}_{kl} | \quad (4.14)$$

the final component field of the linear multiplet.

4.3 Type I vector-tensor multiplet in components

The type I VT multiplet in section 3, possesses the superfield structure given by equations (3.20). Different choices of η_{IJ} lead to the non-linear and linear cases discussed in [9]. However it turns out that the nonlinear case with Chern-Simons terms ($\eta_{1\hat{I}} = 0$) [17]

²⁰The Levi-Civita tensor ε^{mnpq} with world indices is defined as $\varepsilon^{mnpq} := \varepsilon^{abcd} e_a{}^m e_b{}^n e_c{}^p e_d{}^q$.

is equivalent to the most general case by a superfield redefinition [18].²¹ The constraints for this case may be written as [17]

$$\begin{aligned} \nabla_\alpha^{(i} \bar{\nabla}_{\dot{\alpha}}^{j)} L &= 0 , \\ \eta_{11} \nabla^{ij} (\mathcal{Z} L^2) &= 2\eta_{11} \bar{\mathcal{Z}} \bar{\nabla}_{\dot{\alpha}}^{(i} L \bar{\nabla}^{\dot{\alpha}j)} L + \frac{1}{4} \eta_{IJ} [\nabla^{ij} (\frac{\mathcal{W}^I \mathcal{W}^J}{\mathcal{Z}}) - \bar{\nabla}^{ij} (\frac{\bar{\mathcal{W}}^I \bar{\mathcal{W}}^J}{\bar{\mathcal{Z}}})] , \end{aligned} \quad (4.15)$$

with corresponding Lagrangian

$$\begin{aligned} \mathcal{L}^{ij} &= -\frac{1}{6} \eta_{11} L^3 \nabla^{ij} \mathcal{Z} + \eta_{11} \mathcal{Z} L \nabla^i L \nabla^j L + \eta_{11} \bar{\mathcal{Z}} L \bar{\nabla}^i L \bar{\nabla}^j L + \frac{1}{8} \eta_{IJ} [L \nabla^{ij} (\frac{\mathcal{W}^I \mathcal{W}^J}{\mathcal{Z}}) \\ &\quad + L \bar{\nabla}^{ij} (\frac{\bar{\mathcal{W}}^I \bar{\mathcal{W}}^J}{\bar{\mathcal{Z}}}) - 2 \nabla^{ij} (L \frac{\mathcal{W}^I \mathcal{W}^J}{\mathcal{Z}}) - 2 \bar{\nabla}^{ij} (L \frac{\bar{\mathcal{W}}^I \bar{\mathcal{W}}^J}{\bar{\mathcal{Z}}})] . \end{aligned} \quad (4.16)$$

In principle the above constraints and Lagrangian provides everything we need to perform the component reduction and construct the most general action for the type I VT multiplet. In fact, if we are only interested in the bosonic sector then it only requires that we use identities for two spinor derivatives acting on L . However, since the bosonic action is given in [9], here we switch off the Chern-Simons terms (by setting $\eta_{\hat{I}\hat{J}} = 0$) and derive the full action for this case. Finally, to match the normalization in [17] we choose $\eta_{11} = \frac{1}{4}$.

The component fields are defined by

$$\begin{aligned} \ell &:= L| , \quad \Lambda_\alpha^i := \nabla_\alpha^i L| , \quad \bar{\Lambda}_{\dot{\alpha}}^i := \bar{\nabla}_{\dot{\alpha}}^i L| , \quad \ell^{(z)} := \Delta L| , \\ \tilde{v}_m &:= \tilde{\mathcal{V}}_m| , \quad b_{mn} := B_{mn}| , \end{aligned} \quad (4.17)$$

with the component fields of the large vector multiplet reducing to the component fields of the vector multiplet, \mathcal{Z} :

$$\phi = \mathcal{Z}| , \quad \lambda_\alpha^i = \nabla_\alpha^i \mathcal{Z}| , \quad X^{ij} = \nabla^{ij} \mathcal{Z}| , \quad v_m = \mathcal{V}_m| . \quad (4.18)$$

The component field strengths are constructed from the projections of their corre-

²¹The general case may be derived by making the superfield redefinition $L \rightarrow L + c_{\hat{I}} Y^{\hat{I}}$, which in terms of the superfield constraints is equivalent to shifts in the field strength $\mathcal{F} \rightarrow \mathcal{F} + c_{\hat{I}} F^{\hat{I}}$.

spending superspace field strengths

$$\begin{aligned}
f_{mn} &:= F_{mn}| = 2\partial_{[m}v_{n]} , \\
\tilde{f}_{mn} &:= \tilde{\mathcal{F}}_{mn}| = 2(\partial_{[m} + v_{[m}\Delta)\tilde{\mathcal{V}}_{n]}| = 2\partial_{[m}\tilde{v}_{n]} + 2v_{[m}\tilde{f}_{|z|n]} , \\
\tilde{f}_{zm} &:= \Delta\tilde{v}_m \equiv \tilde{v}_m^{(z)} , \\
h_{mnp} &:= 3[(\partial_{[m} + v_{[m}\Delta)B_{np]} - \frac{1}{4}\tilde{\mathcal{V}}_{[m}(\partial_n + V_n\Delta)\tilde{\mathcal{V}}_{p]}]| \\
&= 3[\partial_{[m}b_{np]} - \frac{1}{4}\tilde{v}_{[m}\partial_n\tilde{v}_{p]} + v_{[m}h_{|z|np]}] , \\
h_{zmn} &:= \Delta b_{mn} + \frac{1}{4}\tilde{v}_{[m}\tilde{f}_{|z|n]} .
\end{aligned} \tag{4.19}$$

Their supercovariant field strengths may be found from superspace. Using the superform components in [19] and projecting the identities

$$\begin{aligned}
F_{mn} &= E_m^A E_n^B F_{AB}(-)^{ab} , \quad \tilde{\mathcal{F}}_{mn} = E_m^A E_n^B \tilde{\mathcal{F}}_{AB}(-)^{ab} , \quad \tilde{\mathcal{F}}_{zm} = E_m^A \tilde{\mathcal{F}}_{zA} , \\
H_{mnp} &= E_m^A E_n^B E_p^C H_{ABC}(-)^{ab+ac+bc} , \quad H_{zmn} = E_m^A E_n^B H_{zAB}(-)^{ab} ,
\end{aligned} \tag{4.20}$$

yields results corresponding to the those given in [9].²² However as in [9], we will only need to use some of the supercovariant field strengths:

$$\begin{aligned}
\hat{F}_{ab} &:= F_{ab}| := \frac{1}{2}e_a^m e_b^n f_{mn} - \frac{i}{2}(\sigma_{[a}\alpha^{\dot{\alpha}}\psi_{b]k}\bar{\lambda}_{\dot{\alpha}}^k - \frac{1}{2}\psi_{a\dot{k}}^{\gamma}\psi_{b\gamma}^k\bar{\phi} + \text{c.c.}) , \\
\hat{\mathcal{F}}_{za} &:= \tilde{\mathcal{F}}_{za}| = e_a^m \tilde{v}_m^{(z)} + \frac{i}{2}\psi_{a\dot{i}}^{\alpha}\Lambda_{\alpha}^i - \frac{i}{2}\bar{\psi}_{a\dot{\alpha}}^i\Lambda_{\dot{\alpha}}^{\dot{i}} , \\
\hat{\mathcal{F}}_{ab} &:= \tilde{\mathcal{F}}_{ab}| = \frac{1}{2}e_a^m e_b^n \tilde{f}_{mn} + \frac{1}{2}(\sigma_{[a}\psi_{b]i}^{\alpha}(2\bar{\phi}\bar{\Lambda}_{\dot{\alpha}}^i + \ell\bar{\lambda}_{\dot{\alpha}}^i) + i\psi_{a\dot{k}}^{\gamma}\psi_{b\gamma}^k\bar{\phi}\ell + \text{c.c.})
\end{aligned} \tag{4.21}$$

The reason why this can be done is because the superform components H_{zab} and H_{abc} can be expressed in terms of the field strength components $\tilde{\mathcal{F}}_{ab}$ and $\tilde{\mathcal{F}}_{za}$ respectively (see [19]).

The action is given in terms of the linear multiplet component fields, derived from (4.16). Taking the appropriate covariant derivatives and performing the superspace projection leads to the linear multiplet components:

$$\ell^{ij} = -\frac{1}{24}\ell^3 X^{ij} + \frac{1}{4}\phi\ell\Lambda^i\Lambda^j + \frac{1}{4}\bar{\phi}\ell\bar{\Lambda}^i\bar{\Lambda}^j , \tag{4.22a}$$

²²Our field strengths are defined slightly differently from [9], by an additional term involving $W_{\alpha\beta}$. The component results for the supercovariant field strengths in [9] may also be generated straightforwardly in the general case.

$$\begin{aligned}\chi_{\alpha i} = & -\frac{1}{16}\ell^2 X_{ij}\Lambda_\alpha^j + \frac{i}{12}\ell^3 \nabla_{\alpha\dot{\alpha}}\bar{\lambda}_i^{\dot{\alpha}} + \frac{1}{24}\phi\Lambda_\alpha^j\Lambda_i\Lambda_j \\ & + \frac{1}{8}\bar{\phi}\Lambda_\alpha^j\bar{\Lambda}_i\bar{\Lambda}_j - \frac{i}{2}\ell\hat{\mathcal{F}}_{\alpha\beta}\Lambda_i^\beta - \frac{1}{2}\ell^2\hat{F}_{\alpha\beta}\Lambda_i^\beta\end{aligned}\quad (4.22b)$$

$$\begin{aligned}& -\frac{1}{2}W_{\alpha\beta}\bar{\phi}\ell^2\Lambda_i^\beta + \frac{1}{8}\ell\lambda_i^\beta\Lambda_\beta^j\Lambda_{\alpha j} - \frac{1}{8}\ell\lambda_\alpha^j\Lambda_i\Lambda_j \\ & -\frac{1}{4}\bar{\phi}\phi\ell\ell^{(z)}\Lambda_{\alpha i} + \frac{1}{4}\bar{\phi}\ell(\hat{\mathcal{F}}_{\alpha\dot{\beta},z} - i\nabla_{\alpha\dot{\beta}}\ell)\bar{\Lambda}_i^{\dot{\beta}},\end{aligned}\quad (4.22c)$$

$$\begin{aligned}F = & -\frac{1}{4}\bar{\phi}\ell(\hat{\mathcal{F}}_{az} - i\nabla_a\ell)(\hat{\mathcal{F}}^a_z - i\nabla^a\ell) - \frac{1}{6}\ell^3\Box\bar{\phi} + \frac{1}{2}\bar{\phi}\ell^3D \\ & + \frac{\ell}{2\phi}(\hat{\mathcal{F}}_{\alpha\beta} - i\ell\hat{F}_{\alpha\beta} - i\bar{\phi}\ell W_{\alpha\beta})(\hat{\mathcal{F}}^{\alpha\beta} - i\ell\hat{F}^{\alpha\beta} - i\bar{\phi}\ell W^{\alpha\beta}) - \frac{1}{4}\phi\bar{\phi}^2\ell(\ell^{(z)})^2 \\ & + \frac{L^3}{128\phi}X_{ij}X^{ij} + \frac{L^3}{6}\bar{W}^{\dot{\alpha}\dot{\beta}}(2\bar{F}_{\dot{\alpha}\dot{\beta}}| + \hat{W}_{\dot{\alpha}\dot{\beta}}\bar{\phi}) \\ & + \frac{1}{4}\ell^3\Sigma_{\dot{\alpha}k}\bar{\lambda}^{\dot{\alpha}k} - \frac{\bar{\phi}\ell}{8\phi}\hat{\mathcal{F}}_{\alpha\dot{\alpha},z}\lambda^{\alpha k}\bar{\Lambda}_k^{\dot{\alpha}} - \frac{\ell}{8}\hat{\mathcal{F}}_{\alpha\dot{\alpha},z}\Lambda^{\alpha k}\bar{\lambda}_k^{\dot{\alpha}} - \frac{i\bar{\phi}}{4}\nabla_{\alpha\dot{\alpha}}\ell\Lambda^{\alpha k}\bar{\Lambda}_k^{\dot{\alpha}} \\ & + \frac{3i\ell^2}{16}\nabla_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}k}\Lambda_k^\alpha - \frac{i\bar{\phi}\ell^2}{8\phi}\nabla_{\alpha\dot{\alpha}}\lambda^{\alpha k}\bar{\Lambda}_k^{\dot{\alpha}} + \frac{i\ell}{8}\nabla_{\alpha\dot{\alpha}}\ell\Lambda^{\alpha k}\bar{\lambda}_k^{\dot{\alpha}} - \frac{i\bar{\phi}}{8\phi}\nabla_{\alpha\dot{\alpha}}\ell\lambda^{\alpha k}\bar{\Lambda}_k^{\dot{\alpha}} \\ & - \frac{i\bar{\phi}\ell}{4}\nabla_{\alpha\dot{\alpha}}\Lambda^{\alpha k}\bar{\Lambda}_{\dot{\alpha}k} + \frac{i\bar{\phi}\ell}{4}\Lambda^{\alpha k}\nabla_{\alpha\dot{\alpha}}\bar{\Lambda}_{\dot{\alpha}k} + \frac{\bar{\phi}\ell}{4}\ell^{(z)}\bar{\lambda}_i^{\dot{\alpha}}\bar{\Lambda}_\alpha^i + \frac{\ell^2}{16\phi}X_{ij}\lambda^i\Lambda^j \\ & - \frac{\ell}{16}X_{ij}\Lambda^i\Lambda^j - \frac{\bar{\phi}\ell}{16\phi}X_{ij}\bar{\Lambda}^i\bar{\Lambda}^j + \frac{i\bar{\phi}}{4\phi}(\hat{\mathcal{F}}^{\dot{\alpha}\dot{\beta}} + i\ell\hat{F}^{\dot{\alpha}\dot{\beta}})\bar{\Lambda}_{\dot{\alpha}k}\bar{\Lambda}_{\dot{\beta}}^k \\ & - \frac{i}{4}(\hat{\mathcal{F}}_{\alpha\beta} - 3i\ell\hat{F}_{\alpha\beta} - 2i\bar{\phi}\ell W_{\alpha\beta})\Lambda^{\alpha k}\Lambda_k^\beta + \frac{i\ell}{2\phi}(\hat{\mathcal{F}}_{\alpha\beta} - i\ell\hat{F}_{\alpha\beta} - i\bar{\phi}\ell W_{\alpha\beta})\lambda^{\alpha k}\Lambda_k^\beta \\ & - \frac{i\bar{\phi}\ell}{4\phi}\nabla_{\alpha\dot{\alpha}}\phi\Lambda^{\alpha k}\bar{\Lambda}_k^{\dot{\alpha}} + \frac{i}{4}\ell\nabla_{\alpha\dot{\alpha}}\bar{\phi}\Lambda^{\alpha k}\bar{\Lambda}_k^{\dot{\alpha}} \\ & + \frac{\ell}{8\phi}\lambda^i\Lambda^j\lambda_i\Lambda_j + \frac{\ell}{8\phi}\lambda^k\lambda^l\Lambda_k\Lambda_l - \frac{\bar{\phi}}{8\phi}\bar{\lambda}_i\bar{\Lambda}_j\bar{\Lambda}^i\bar{\Lambda}^j - \frac{1}{12}\lambda^i\Lambda^j\Lambda_i\Lambda_j - \frac{\bar{\phi}}{8\phi}\lambda^i\Lambda^j\bar{\Lambda}_i\bar{\Lambda}_j \\ & - \frac{\bar{\phi}}{8\phi}\bar{\lambda}_i\bar{\Lambda}_j\bar{\Lambda}^i\bar{\Lambda}^j - \frac{\phi}{32\ell}\Lambda_i\Lambda_j\Lambda^i\Lambda^j - \frac{\bar{\phi}^2}{32\phi\ell}\bar{\Lambda}_i\bar{\Lambda}_j\bar{\Lambda}^i\bar{\Lambda}^j + \frac{\bar{\phi}}{16\ell}\Lambda_i\Lambda_j\bar{\Lambda}^i\bar{\Lambda}^j,\end{aligned}\quad (4.22d)$$

$$\begin{aligned}\mathfrak{E}_a = & \frac{1}{2}\phi\bar{\phi}\ell\ell^{(z)}\nabla_a\ell - \frac{1}{2}(\ell\hat{\mathcal{F}}^b_z + i\ell\nabla^b\ell + \frac{1}{2}(\sigma^b)^\gamma{}_{\dot{\gamma}}\Lambda_\gamma^k\bar{\Lambda}_{\dot{\gamma}}^k)(\hat{\mathcal{F}}_{ab}^+ - i\ell\hat{F}_{ab}^+ - iW_{ab}^+\bar{\phi}\ell) \\ & - \frac{1}{2}(\ell\hat{\mathcal{F}}^b_z - i\ell\nabla^b\ell + \frac{1}{2}(\sigma^b)^\gamma{}_{\dot{\gamma}}\Lambda_\gamma^k\bar{\Lambda}_{\dot{\gamma}}^k)(\hat{\mathcal{F}}_{ab}^- + i\ell\hat{F}_{ab}^- + iW_{ab}^-\phi\ell) \\ & - \frac{1}{6}\ell^3\nabla^b(2\hat{F}_{ab}^- + W_{ab}^-\phi) - \frac{1}{6}\ell^3\nabla^b(W_{ab}^+\bar{\phi}) - \frac{i}{8}(\sigma_a)_\alpha{}^{\dot{\alpha}}\ell^3\Sigma^{\alpha k}\bar{\lambda}_{\dot{\alpha}k} - \frac{i}{16}(\sigma^a)^{\alpha\dot{\alpha}}X_{ij}\Lambda_\alpha^i\bar{\Lambda}_{\dot{\alpha}}^j \\ & + \frac{1}{4}\bar{\phi}\ell\nabla_a\Lambda_\gamma^k\bar{\Lambda}_{\dot{\gamma}}^k - \frac{1}{4}\phi\ell\nabla_a\Lambda_\gamma^k\bar{\Lambda}_{\dot{\gamma}}^k + \frac{1}{4}(\sigma_{ab})^{\alpha\beta}\ell\nabla^b\phi\Lambda_\alpha^k\Lambda_{\beta k} - \frac{1}{4}(\tilde{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}}\ell\nabla^b\bar{\phi}\bar{\Lambda}_{\dot{\alpha}k}\bar{\Lambda}_{\dot{\beta}}^k \\ & - \frac{1}{8}\ell^2\nabla_a\lambda_\gamma^k\Lambda_k^\gamma + \frac{1}{8}\ell^2\nabla_a\bar{\lambda}_{\dot{\gamma}}^k\bar{\Lambda}_k^{\dot{\gamma}} + \frac{1}{4}(\sigma_{ab})^{\alpha\beta}\ell^2\nabla^b\lambda_\alpha^k\Lambda_{\beta k} - \frac{1}{4}(\tilde{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}}\ell^2\nabla^b\bar{\lambda}_{\dot{\alpha}k}\bar{\Lambda}_{\dot{\beta}}^k \\ & + \frac{i\ell}{8}(\hat{\mathcal{F}}_{az} + i\nabla_a\ell)\lambda_\gamma^k\Lambda_k^\gamma + \frac{i\ell}{8}(\hat{\mathcal{F}}_{az} - i\nabla_a\ell)\bar{\lambda}_{\dot{\gamma}}^k\bar{\Lambda}_k^{\dot{\gamma}}\end{aligned}$$

$$\begin{aligned}
& -\frac{i\ell}{4}(\sigma_{ab})^{\alpha\beta}(\hat{\mathcal{F}}^b_z + i\nabla^b\ell)\lambda^k_\alpha\Lambda_{\beta k} - \frac{i\ell}{4}(\tilde{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}}(\hat{\mathcal{F}}^b_z - i\nabla^b\ell)\bar{\lambda}^{\dot{\alpha}}_k\bar{\Lambda}^{\dot{\beta}k} \\
& -\frac{i}{16}(\sigma_a)^{\alpha\dot{\alpha}}\lambda_i\Lambda_j\Lambda^j_\alpha\bar{\Lambda}^i_{\dot{\alpha}} - \frac{i}{16}(\sigma^a)^{\alpha\dot{\alpha}}\bar{\lambda}^i\bar{\Lambda}^j\Lambda_{\alpha i}\bar{\Lambda}_{\dot{\alpha}j} \\
& -\frac{i}{16}(\sigma^a)^{\alpha\dot{\alpha}}\lambda^i_\alpha\bar{\Lambda}^j_{\dot{\alpha}}\Lambda_i\Lambda_j + \frac{i}{16}\bar{\lambda}^i_\alpha\Lambda^j_\alpha\bar{\Lambda}_i\bar{\Lambda}_j, \tag{4.22e}
\end{aligned}$$

where for compactness we have defined a number of composite objects. We define for terms arising from vector derivatives of the vector multiplet fields:

$$\nabla_a\phi \equiv \nabla'_a\phi - \frac{1}{2}(\psi_{aj}\lambda^j), \tag{4.23a}$$

$$\nabla_b\bar{\lambda}^{\dot{\alpha}}_j \equiv \nabla'_b\bar{\lambda}^{\dot{\alpha}}_j + 2\bar{\phi}\bar{\phi}_{bj}^{\dot{\alpha}} - i\psi_{b\beta j}\nabla^{\dot{\alpha}\beta}\bar{\phi} + \frac{1}{4}\bar{\psi}_b^{\dot{\alpha}k}X_{kj} + 2\bar{\psi}_{b\beta j}\hat{F}^{\dot{\beta}\alpha} + \bar{\psi}_{b\beta j}\bar{W}^{\dot{\beta}\alpha}\phi, \tag{4.23b}$$

$$\begin{aligned}
\Box\bar{\phi} & \equiv \nabla'^a\nabla_a\bar{\phi} + 2\mathfrak{f}_a^a\bar{\phi} - \frac{i}{2}(\phi_m^j\sigma^m\bar{\lambda}_j) \\
& + \frac{i}{4}(\psi_{mj}\sigma^m)_{\dot{\alpha}}\bar{W}^{\dot{\alpha}\beta}\bar{\lambda}^j_{\dot{\beta}} - \frac{3i}{4}(\psi_{mj}\sigma^m\bar{\Sigma}^j)\bar{\phi} - \frac{1}{2}\bar{\psi}_{a\dot{\beta}}^j\nabla^a\bar{\lambda}^{\dot{\beta}}_j, \tag{4.23c}
\end{aligned}$$

$$\begin{aligned}
\nabla_a\hat{F}_{\alpha\beta} & = \nabla'_a\hat{F}_{\alpha\beta} - \phi_{a(\alpha}\lambda_{\beta)k} + \frac{1}{4}\psi_{a(\alpha k}(i\nabla_{\beta)}^{\dot{\gamma}}\bar{\lambda}^k_{\dot{\gamma}} + 2\Sigma_{\beta)}^k\bar{\phi}) \\
& - \frac{1}{4}\psi_{ak}^{\dot{\gamma}}(2\Psi_{(\alpha\beta\dot{\gamma})} + i\Psi_{(\alpha\dot{\alpha}}^{\dot{\alpha}k}W_{\beta\dot{\gamma}}))\bar{\phi} - \bar{\psi}_{a\dot{\gamma}}^k(i\nabla_{(\alpha}^{\dot{\gamma}}\lambda_{\beta)k} - W_{\alpha\beta}\bar{\lambda}^{\dot{\gamma}}_k), \tag{4.23d}
\end{aligned}$$

as well as for the vector-tensor fields

$$\nabla_a\ell \equiv \nabla'_a\ell - \frac{1}{2}(\psi_{aj}\Lambda^j) - \frac{1}{2}(\bar{\psi}_a^j\bar{\Lambda}_j), \tag{4.24a}$$

$$\begin{aligned}
\nabla_a\Lambda^i_\alpha & \equiv \nabla'_a\Lambda^i_\alpha - \frac{1}{2}\bar{\psi}_a^i\hat{\mathcal{F}}_{\alpha\dot{\beta}}^{\dot{\beta}}(z) + i\nabla_{\alpha}^{\dot{\beta}}\ell \\
& + \frac{1}{4}\psi_{a\alpha j}\left(-\frac{1}{\ell}\Lambda^i\Lambda^j + \frac{\bar{\phi}}{\phi\ell}\bar{\Lambda}^i\bar{\Lambda}^j - \frac{2}{\phi}\lambda^{(i}\Lambda^{j)} - \frac{\ell}{2\phi}X^{ij}\right) \\
& - \frac{1}{2}\psi_a^{\beta i}\left(\frac{2i}{\phi}\hat{\mathcal{F}}_{\alpha\beta} + \frac{2\ell}{\phi}\hat{F}_{\alpha\beta} + \frac{2\bar{\phi}}{\phi}W_{\alpha\beta} - \frac{1}{\phi}\lambda_{(\alpha}\Lambda_{\beta)}\right) + \frac{1}{2}\psi_a^i\bar{\phi}\ell^{(z)}. \tag{4.24b}
\end{aligned}$$

We also define

$$\nabla_a W_{\alpha\beta} = \nabla'_a W_{\alpha\beta}. \tag{4.25}$$

The derivative $\nabla'_a = \nabla'_a + v_a\Delta$, with ∇'_a given in appendix B, is covariant with respect to Lorentz, dilatation, $U(1)_R$ and $SU(2)_R$ transformations. Finally, from the constraints one can derive the following useful identity

$$\begin{aligned}
\phi\Delta\Lambda^i_\alpha & = i\nabla_{\alpha\dot{\alpha}}\bar{\Lambda}^{\dot{\alpha}i} - \lambda^i_\alpha\ell^{(z)} - \frac{1}{4\ell^2}\Lambda_{\alpha j}\bar{\Lambda}^i\bar{\Lambda}^j - \frac{1}{2\ell}\hat{\mathcal{F}}_{\alpha\dot{\alpha},z}\bar{\Lambda}^{\dot{\alpha}i} \\
& + \frac{i}{2\ell}\nabla_{\alpha\dot{\alpha}}\bar{\Lambda}^{\dot{\alpha}i} + \frac{\phi}{4\bar{\phi}\ell^2}\Lambda_{\alpha j}\Lambda^i\Lambda^j - \frac{i}{\bar{\phi}\ell}\hat{\mathcal{F}}_{\alpha\beta}\Lambda^{\beta i} - \frac{1}{\bar{\phi}}\hat{F}_{\alpha\beta}\Lambda^{\beta i} \\
& - W_{\alpha\beta}\Lambda^{\beta i} - \frac{\phi}{2\ell}\ell^{(z)}\Lambda^i_\alpha + \frac{i}{\bar{\phi}}\nabla_{\alpha\dot{\beta}}\bar{\phi}\bar{\Lambda}^{\dot{\beta}i} - \frac{1}{2\bar{\phi}}\hat{\mathcal{F}}_{\alpha\dot{\beta},z}\bar{\Lambda}^{\dot{\beta}i}
\end{aligned}$$

$$\begin{aligned}
& + \frac{i}{2\bar{\phi}} \nabla_{\alpha\dot{\beta}} \ell \bar{\lambda}^{\dot{\beta}i} + \frac{1}{8\bar{\phi}} X^{ij} \Lambda_{\alpha j} + \frac{i\ell}{2\bar{\phi}} \nabla_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}i} \\
& + \frac{1}{4\bar{\phi}\ell} \lambda^{\beta i} \Lambda_{\alpha}^j \Lambda_{\beta j} + \frac{1}{4\bar{\phi}\ell} \lambda_{\alpha j} \Lambda^i \Lambda^j ,
\end{aligned} \tag{4.26}$$

which may be used to make explicit the terms containing v_a .

The bosonic sector of the above results can be shown to agree with [9].²³

4.4 Type II vector-tensor multiplet in components

The type II VT multiplet possesses the superfield structure given by equations (3.23). The corresponding Lagrangian was constructed in [19]

$$\mathcal{L}^{ij} = \frac{i}{2} e^{-iL} \nabla^{ij} (\mathcal{Z} K L e^{2iL}) - e^{-iL} \bar{\mathcal{Z}} \bar{K} \bar{\nabla}^i L \bar{\nabla}^j L - \frac{1}{4} e^{-iL} \bar{\nabla}^{ij} (\bar{\mathcal{Z}} \bar{K}) + \text{c.c.} \tag{4.27}$$

To simplify the component reduction we will look at the pure case where $\eta_{11} = 1$, $\eta_{1\hat{I}} = \eta_{\hat{I}j} = 0$, which represents a supergravity extension of the VT multiplet of Theis [25]. In this case the constraints may be written as

$$\nabla_{\alpha}^{(i} \bar{\nabla}_{\dot{\alpha}}^{j)} L = 0 , \tag{4.28}$$

$$\begin{aligned}
\nabla^{ij} L = & - \frac{2}{\tan 2L} \nabla^i L \nabla^j L - \frac{2\bar{\mathcal{Z}}}{\mathcal{Z} \sin 2L} \bar{\nabla}^i L \bar{\nabla}^j L - \frac{2}{\mathcal{Z}} \nabla^{(i} \mathcal{Z} \nabla^{j)} L \\
& + \frac{1}{2\mathcal{Z} \tan L} \nabla^{ij} \mathcal{Z} ,
\end{aligned} \tag{4.29}$$

with corresponding Lagrangian

$$\begin{aligned}
\mathcal{L}^{ij} = & - 2e^{iL} \mathcal{Z} \nabla^i L \nabla^j L - 2e^{-iL} \bar{\mathcal{Z}} \bar{\nabla}^i L \bar{\nabla}^j L - \cos L \nabla^{ij} \mathcal{Z} + iLe^{-iL} \nabla^{ij} (\mathcal{Z} e^{2iL}) \\
= & 2\mathcal{Z} \left(\frac{2Le^{-iL}}{\sin 2L} - e^{iL} \right) \nabla^i L \nabla^j L + 2\bar{\mathcal{Z}} \left(\frac{2Le^{iL}}{\sin 2L} - e^{-iL} \right) \bar{\nabla}^i L \bar{\nabla}^j L \\
& - \left(\cos L + \frac{L}{\sin L} \right) \nabla^{ij} \mathcal{Z} .
\end{aligned} \tag{4.30}$$

The component fields are

$$\begin{aligned}
\ell & := L| , \quad \Lambda_{\alpha}^i := \nabla_{\alpha}^i L| , \quad \bar{\Lambda}_{\dot{\alpha}}^i := \bar{\nabla}_{\dot{\alpha}}^i L| , \quad \ell^{(z)} := \Delta L| , \\
v_m & := \mathcal{V}_m| , \quad b_{mn} := B_{mn}| ,
\end{aligned} \tag{4.31}$$

²³There is however a conventional difference, which changes where the auxiliary field D emerges (see [23]).

with the component fields of the vector multiplet, \mathcal{Z} , defined by²⁴

$$\tilde{\phi} = \mathcal{Z}|, \quad \tilde{\lambda}_\alpha^i = \nabla_\alpha^i \mathcal{Z}|, \quad \tilde{X}^{ij} = \nabla^{ij} \mathcal{Z}|, \quad \tilde{v}_m = \tilde{V}_m|. \quad (4.32)$$

The component field strengths and their corresponding supercovariant field strengths are constructed as in the previous subsection. The component field strengths are

$$\begin{aligned} \tilde{f}_{mn} &:= F_{mn}| = 2\partial_{[m}\tilde{v}_{n]} , \\ f_{mn} &:= \mathcal{F}_{mn}| = 2(\partial_{[m} + v_{[m}\Delta)\mathcal{V}_{n]}| = 2\partial_{[m}\tilde{v}_{n]} + 2v_{[m}f_{zn]} , \\ f_{zm} &:= \Delta v_m \equiv v_m^{(z)} , \\ h_{mnp} &:= 3[(\partial_{[m} + v_{[m}\Delta)B_{np]} - \frac{1}{4}\tilde{\mathcal{V}}_{[m}(\partial_n + V_n\Delta)\tilde{\mathcal{V}}_{p]}]| \\ &= 3[\partial_{[m}b_{np]} - \frac{1}{4}\tilde{v}_{[m}\partial_n\tilde{v}_{p]} + v_{[m}h_{|z|np]}] , \\ h_{zmn} &:= \Delta b_{mn} , \end{aligned} \quad (4.33)$$

with corresponding supercovariant field strengths

$$\begin{aligned} \hat{F}_{ab} &:= F_{ab}| := \frac{1}{2}e_a^m e_b^n \tilde{f}_{mn} - \frac{i}{2}(\sigma_{[a})_\alpha{}^{\dot{\alpha}}\psi_{b]}^{\dot{\alpha}}\tilde{\lambda}_\alpha^k - \frac{1}{2}\psi_a^\gamma\psi_b^k\tilde{\phi} + \text{c.c.} , \\ \hat{\mathcal{F}}_{za} &:= \mathcal{F}_{za}| = e_a^m v_m^{(z)} - \frac{i}{2}\psi_{a\dot{\alpha}}\Lambda_{\dot{\alpha}}^i + \frac{i}{2}\bar{\psi}_{a\dot{\alpha}}\bar{\Lambda}_{\dot{\alpha}}^i , \\ \hat{\mathcal{F}}_{ab} &:= \mathcal{F}_{ab}| = \frac{1}{2}e_a^m e_b^n f_{mn} + \frac{1}{2}(\sigma_{[a})_\alpha{}^{\dot{\alpha}}\psi_{b]}^{\dot{\alpha}}(\tilde{\lambda}_\alpha^i + 2i\bar{\Lambda}_{\dot{\alpha}}^i)e^{i\ell} + i\psi_a^\gamma\psi_b^k\tilde{\phi}e^{i\ell} + \text{c.c.} , \\ \hat{H}_{zab} &:= \hat{H}_{zab}| = \frac{1}{2}e_a^m e_b^n h_{zmn} - \frac{1}{4}(\sigma_{[a})_\alpha{}^{\dot{\alpha}}\psi_{b]}^{\dot{\alpha}}(\tilde{\lambda}_\alpha^i - 2i\bar{\Lambda}_{\dot{\alpha}}^i)e^{-i\ell} - \frac{i}{2}\bar{\psi}_{a\dot{\alpha}}\bar{\psi}_{b\dot{\alpha}}\tilde{\phi}e^{-i\ell} + \text{c.c.} , \\ \hat{H}_{abc} &:= H_{abc}| = \frac{1}{2}e_a^m e_b^n e_c^p h_{mnp} + \frac{3}{4}(\sigma_{[ab})_\alpha{}^{\dot{\alpha}}\psi_{c]}^{\dot{\alpha}}\tilde{\phi}\left((\cos 2\ell + 1)\tilde{\lambda}_\alpha^i - 2\tilde{\phi}\sin(2\ell)\Lambda_\alpha^i\right) \\ &\quad + \frac{3}{4}(\sigma_{[a})_\alpha{}^{\dot{\alpha}}\psi_{b\dot{\alpha}}\bar{\psi}_{c\dot{\alpha}}\tilde{\phi}\tilde{\phi}(\cos 2\ell + 1) + \text{c.c.} \end{aligned} \quad (4.34)$$

Similar to the previous case we will only need the expressions for \hat{F}_{ab} , $\hat{\mathcal{F}}_{za}$ and $\hat{\mathcal{F}}_{ab}$.

The type II VT multiplet possesses more complicated component structure, which is highlighted by the appearance of trigonometric functions. Therefore for simplicity sake we provide results for the bosonic sector of the action. The component fields of the linear

²⁴The tildes signify the differing roles \mathcal{Z} play for the two types of VT multiplets.

multiplet are:

$$\begin{aligned}\ell^{ij} &= 2\tilde{\phi}\left(\frac{2\ell e^{-i\ell}}{\sin 2\ell} - e^{i\ell}\right)\Lambda^i\Lambda^j + 2\tilde{\bar{\phi}}\left(\frac{2\ell e^{i\ell}}{\sin 2\ell} - e^{-i\ell}\right)\bar{\Lambda}^i\bar{\Lambda}^j \\ &\quad - \left(\cos \ell + \frac{\ell}{\sin \ell}\right)X^{ij},\end{aligned}\tag{4.35a}$$

$$\begin{aligned}\chi_{\alpha i} &= \left(-\frac{2e^{2i\ell}+1}{4\sin \ell} + \frac{\ell e^{-i\ell}}{2\sin^2 \ell}\right)X_{ij}\Lambda_{\alpha}^j \\ &\quad - 2\left(\frac{2\ell e^{-i\ell}}{\sin 2\ell} - e^{i\ell}\right)\left(2i\hat{F}_{\alpha\beta} + iW_{\alpha\beta}\tilde{\bar{\phi}}(1+e^{2i\ell}) - 2e^{i\ell}\hat{\mathcal{F}}_{\alpha\beta}\right)\Lambda_i^{\beta} \\ &\quad + 2i\tilde{\phi}\tilde{\bar{\phi}}e^{i\ell}\left(\frac{2\ell e^{-i\ell}}{\sin 2\ell} - e^{i\ell}\right)\ell^{(z)}\Lambda_{\alpha i} \\ &\quad - 2\tilde{\bar{\phi}}\left(\frac{2\ell e^{i\ell}}{\sin 2\ell} - e^{-i\ell}\right)(\hat{\mathcal{F}}_{\alpha\dot{\alpha},z} + i\nabla'_{\alpha\dot{\alpha}}\ell)\bar{\Lambda}_i^{\dot{\alpha}} \\ &\quad + 2i(\cos \ell + \frac{\ell}{\sin \ell})\nabla_{\alpha\dot{\alpha}}\bar{\nabla}_i^{\dot{\alpha}}\tilde{\bar{\phi}} + 3 \text{ fermion terms},\end{aligned}\tag{4.35b}$$

$$\begin{aligned}F &= \frac{1}{32\tilde{\phi}\sin \ell \tan \ell}\left(\frac{2\ell e^{-i\ell}}{\sin \ell} - 2e^{2i\ell} + 1\right)\tilde{X}^{ij}\tilde{X}_{ij} \\ &\quad - \frac{1}{\tilde{\phi}}\left(\frac{2\ell e^{-i\ell}}{\sin 2\ell} - e^{i\ell}\right)(2i\hat{F}_{\alpha\beta} + iW_{\alpha\beta}\tilde{\bar{\phi}}(1+e^{2i\ell}) - 2e^{i\ell}\hat{\mathcal{F}}_{\alpha\beta}) \\ &\quad \quad \times (2i\hat{F}^{\alpha\beta} + iW^{\alpha\beta}\tilde{\bar{\phi}}(1+e^{2i\ell}) - 2e^{i\ell}\hat{\mathcal{F}}^{\alpha\beta}) \\ &\quad + 2\tilde{\phi}\tilde{\bar{\phi}}^2e^{2i\ell}\left(\frac{2\ell e^{-i\ell}}{\sin 2\ell} - e^{i\ell}\right)(\ell^{(z)})^2 \\ &\quad - 2\tilde{\bar{\phi}}\left(\frac{2\ell e^{i\ell}}{\sin 2\ell} - e^{-i\ell}\right)(\hat{\mathcal{F}}_{az} + i\nabla'_a\ell)(\hat{\mathcal{F}}^a_z + i\nabla'^a\ell) \\ &\quad - 4(\cos \ell + \frac{\ell}{\sin \ell})\square\tilde{\bar{\phi}} + 4(\cos \ell + \frac{\ell}{\sin \ell})\bar{W}_{\dot{\alpha}\dot{\beta}}(2\hat{F}^{\dot{\alpha}\dot{\beta}} + \bar{W}^{\dot{\alpha}\dot{\beta}}\tilde{\bar{\phi}}) \\ &\quad + 12\tilde{\bar{\phi}}(\cos \ell + \frac{\ell}{\sin \ell})D + \text{fermion terms},\end{aligned}\tag{4.35c}$$

$$\begin{aligned}\mathfrak{E}_a &= -2i\left(\frac{2\ell e^{-i\ell}}{\sin 2\ell} - e^{i\ell}\right)(2i\hat{F}_{ab}^+ + iW_{ab}^+\tilde{\bar{\phi}}(1+e^{2i\ell}) - 2e^{i\ell}\hat{\mathcal{F}}_{ab}^+)(\hat{\mathcal{F}}^b_z - i\nabla'^b\ell) \\ &\quad - 2i\left(\frac{2\ell e^{i\ell}}{\sin 2\ell} - e^{-i\ell}\right)(2i\hat{F}_{ab}^- + iW_{ab}^-\tilde{\bar{\phi}}(1+e^{-2i\ell}) - 2e^{-i\ell}\hat{\mathcal{F}}_{ab}^-)(\hat{\mathcal{F}}^b_z + i\nabla'^b\ell) \\ &\quad + 4\tilde{\phi}\tilde{\bar{\phi}}\left(\frac{2\ell}{\sin 2\ell} - \cos 2\ell\right)\ell^{(z)}\hat{\mathcal{F}}_{az} - 4\tilde{\phi}\tilde{\bar{\phi}}\sin(2\ell)\ell^{(z)}\nabla'_a\ell \\ &\quad - 4(\cos \ell + \frac{\ell}{\sin \ell})\nabla^b(2\hat{F}_{ab}^- + W_{ab}^-\tilde{\bar{\phi}}) - 4(\cos \ell + \frac{\ell}{\sin \ell})\nabla^b(W_{ab}^+\tilde{\bar{\phi}}) \\ &\quad + \text{fermion terms}.\end{aligned}\tag{4.35d}$$

The composite objects are defined as in the previous section, but with the component fields of \mathcal{Z} defined with a tilde. Finally the component fields of the large vector multiplet

appearing in the action principle (4.9) are

$$\begin{aligned}\phi &= i\tilde{\phi}e^{-i\ell} , \quad \lambda_{\alpha}^i = ie^{-i\ell}\tilde{\lambda}_{\alpha}^i + 2e^{-i\ell}\tilde{\phi}\Lambda_{\alpha}^i , \\ X^{ij} &= \frac{1}{\sin \ell}\tilde{X}^{ij} - 4\frac{e^{i\ell}}{\sin 2\ell}\tilde{\phi}\Lambda^i\Lambda^j - 4\frac{e^{-i\ell}}{\sin 2\ell}\tilde{\phi}\bar{\Lambda}^i\bar{\Lambda}^j .\end{aligned}\tag{4.36}$$

5 Conclusion

One of the main goals of this paper was to exhaust possible versions of the VT multiplet in conformal superspace. This was motivated by the numerous papers exploring possible deformations of the original VT multiplet. Generalizing the superform formulations of [19] to a suitable framework for all known VT multiplets, we found two distinct cases. Interestingly, these two cases correspond to generalizations of the possible deformations in flat superspace. Moreover, the possibility of making use of the variant VT multiplet to construct an alternative off-shell formulation for the VT multiplet of [9] was ruled out.

The second main goal was to construct component actions for VT multiplets using superspace techniques. We found the construction of the component action from a superfield Lagrangian to be efficient, with superform formulations facilitating the component reduction. Most notably the full component action for the standard non-linear VT multiplet was derived together with the bosonic sector for the supergravity extension of the VT multiplet of Theis [25]. These represent completely new results and it would be interesting if the type II VT multiplet turns out to have applications in string theory. If this turns out to be the case then our superform formulation would be indispensable.

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A Conformal superspace

This appendix contains a brief summary of conformal superspace of [23].²⁵ Consider a curved 4D $\mathcal{N} = 2$ superspace $\mathcal{M}^{4|8}$ parametrized by local bosonic (x) and fermionic $(\theta, \bar{\theta})$ coordinates $z^M = (x^m, \theta_{\dot{\iota}}^{\mu}, \bar{\theta}_{\dot{\mu}}^{\dot{\iota}})$, where $m = 0, 1, \dots, 3$, $\mu = 1, 2$, $\dot{\mu} = 1, 2$ and $\dot{\iota} = \underline{1}, \underline{2}$.

²⁵We use the conventions of [18], which follow closely the conventions of [44].

The Grassmann variables θ_i^μ and $\bar{\theta}_\mu^i$ are related to each other by complex conjugation: $\overline{\theta_i^\mu} = \bar{\theta}^{\mu i}$. The covariant derivatives $\nabla_A = (\nabla_a, \nabla_\alpha^i, \bar{\nabla}_i^{\dot{\alpha}})$ have the form

$$\begin{aligned}\nabla_A &= E_A + \frac{1}{2}\Omega_A^{ab}M_{ab} + \Phi_A^{ij}J_{ij} + i\Phi_A Y + B_A\mathbb{D} + \mathfrak{F}_A^B K_B \\ &= E_A + \Omega_A^{\beta\gamma}M_{\beta\gamma} + \bar{\Omega}_A^{\dot{\beta}\dot{\gamma}}\bar{M}_{\dot{\beta}\dot{\gamma}} + \Phi_A^{ij}J_{ij} + i\Phi_A Y + B_A\mathbb{D} + \mathfrak{F}_A^B K_B .\end{aligned}\quad (\text{A.1})$$

Here $E_A = E_A^M(z)\partial_M$ is the supervielbein, with $\partial_M = \partial/\partial z^M$, $J_{kl} = J_{lk}$ are generators of the group $\text{SU}(2)_R$, M_{ab} are the Lorentz generators, Y is the generator of the chiral rotation group $\text{U}(1)_R$, and $K^A = (K^a, S_i^\alpha, \bar{S}_{\dot{\alpha}}^i)$ are the special superconformal generators. The one-forms Ω_A^{bc} , Φ_A^{kl} , Φ_A , B_A and \mathfrak{F}_A^B are the corresponding connections.

The generators act on the covariant derivatives as

$$\begin{aligned}[M_{ab}, \nabla_c] &= 2\eta_{c[a}\nabla_{b]} , \quad [M_{ab}, \nabla_\alpha^i] = (\sigma_{ab})_\alpha^\beta \nabla_\beta^i , \quad [M_{ab}, \bar{\nabla}_i^{\dot{\alpha}}] = (\tilde{\sigma}_{ab})^{\dot{\alpha}}_{\dot{\beta}} \bar{\nabla}_i^{\dot{\beta}} , \\ [J_{ij}, \nabla_\alpha^k] &= -\delta_{(i}^k \nabla_{\alpha j)} , \quad [J_{ij}, \bar{\nabla}_k^{\dot{\alpha}}] = -\varepsilon_{k(i} \bar{\nabla}_{j)}^{\dot{\alpha}} , \\ [Y, \nabla_\alpha^i] &= \nabla_\alpha^i , \quad [Y, \bar{\nabla}_i^{\dot{\alpha}}] = -\bar{\nabla}_i^{\dot{\alpha}} , \\ [\mathbb{D}, \nabla_a] &= \nabla_a , \quad [\mathbb{D}, \nabla_\alpha^i] = \frac{1}{2}\nabla_\alpha^i , \quad [\mathbb{D}, \bar{\nabla}_i^{\dot{\alpha}}] = \frac{1}{2}\bar{\nabla}_i^{\dot{\alpha}} .\end{aligned}\quad (\text{A.2})$$

Finally, the algebra of K^A with ∇_B is given by

$$\begin{aligned}[K^a, \nabla_b] &= 2\delta_b^a \mathbb{D} + 2M^a_b , \\ \{S_i^\alpha, \nabla_\beta^j\} &= 2\delta_i^j \delta_\beta^\alpha \mathbb{D} - 4\delta_i^j M^\alpha_\beta - \delta_i^j \delta_\beta^\alpha Y + 4\delta_\beta^\alpha J_i^j , \\ \{\bar{S}_{\dot{\alpha}}^i, \bar{\nabla}_j^{\dot{\beta}}\} &= 2\delta_j^i \delta_{\dot{\alpha}}^{\dot{\beta}} \mathbb{D} + 4\delta_j^i \bar{M}_{\dot{\alpha}}^{\dot{\beta}} + \delta_j^i \delta_{\dot{\alpha}}^{\dot{\beta}} Y - 4\delta_{\dot{\alpha}}^{\dot{\beta}} J_j^i , \\ [K^a, \nabla_\beta^j] &= -i(\sigma^a)_\beta^{\dot{\beta}} \bar{S}_{\dot{\beta}}^j , \quad [K^a, \bar{\nabla}_j^{\dot{\beta}}] = -i(\sigma^a)^{\dot{\beta}}_\beta S_j^\beta , \\ [S_i^\alpha, \nabla_b] &= i(\sigma_b)^\alpha_{\dot{\beta}} \bar{\nabla}_i^{\dot{\beta}} , \quad [\bar{S}_{\dot{\alpha}}^i, \nabla_b] = i(\sigma_b)_{\dot{\alpha}}^\beta \nabla_b^i ,\end{aligned}\quad (\text{A.3})$$

where all other (anti-)commutations vanish.

The covariant derivatives obey the (anti-)commutation relations:

$$\{\nabla_\alpha^i, \nabla_\beta^j\} = 2\varepsilon^{ij}\varepsilon_{\alpha\beta}\bar{W}_{\dot{\gamma}\dot{\delta}}\bar{M}^{\dot{\gamma}\dot{\delta}} + \frac{1}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta}\bar{\nabla}_{\dot{\gamma}k}\bar{W}^{\dot{\gamma}\dot{\delta}}\bar{S}_{\dot{\delta}}^k - \frac{1}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta}\nabla_{\gamma\dot{\delta}}\bar{W}^{\dot{\delta}}_{\dot{\gamma}}K^{\gamma\dot{\gamma}} , \quad (\text{A.4a})$$

$$\{\bar{\nabla}_i^{\dot{\alpha}}, \bar{\nabla}_j^{\dot{\beta}}\} = -2\varepsilon_{ij}\varepsilon^{\dot{\alpha}\dot{\beta}}W^{\gamma\delta}M_{\gamma\delta} + \frac{1}{2}\varepsilon_{ij}\varepsilon^{\dot{\alpha}\dot{\beta}}\nabla^{\gamma k}W_{\gamma\delta}S_k^\delta - \frac{1}{2}\varepsilon_{ij}\varepsilon^{\dot{\alpha}\dot{\beta}}\nabla^{\gamma\dot{\gamma}}W_{\gamma}{}^\delta K_{\delta\dot{\gamma}} , \quad (\text{A.4b})$$

$$\{\nabla_\alpha^i, \bar{\nabla}_j^{\dot{\beta}}\} = -2i\delta_j^i \nabla_\alpha^{\dot{\beta}} , \quad (\text{A.4c})$$

$$\begin{aligned}
[\nabla_{\alpha\dot{\alpha}}, \nabla_{\beta}^i] &= -i\varepsilon_{\alpha\beta}\bar{W}_{\dot{\alpha}\dot{\beta}}\bar{\nabla}^{\dot{\beta}i} - \frac{i}{2}\varepsilon_{\alpha\beta}\bar{\nabla}^{\dot{\beta}i}\bar{W}_{\dot{\alpha}\dot{\beta}}\mathbb{D} - \frac{i}{4}\varepsilon_{\alpha\beta}\bar{\nabla}^{\dot{\beta}i}\bar{W}_{\dot{\alpha}\dot{\beta}}Y + i\varepsilon_{\alpha\beta}\bar{\nabla}_{\dot{j}}^{\dot{\beta}}\bar{W}_{\dot{\alpha}\dot{\beta}}J^{ij} \\
&\quad - i\varepsilon_{\alpha\beta}\bar{\nabla}_{\dot{\beta}}^i\bar{W}_{\dot{\gamma}\dot{\alpha}}\bar{M}^{\dot{\beta}\dot{\gamma}} - \frac{i}{4}\varepsilon_{\alpha\beta}\bar{\nabla}_{\dot{\alpha}}^i\bar{\nabla}_{\dot{k}}^{\dot{\beta}}\bar{W}_{\dot{\beta}\dot{\gamma}}\bar{S}^{\dot{\gamma}k} + \frac{1}{2}\varepsilon_{\alpha\beta}\nabla^{\gamma\dot{\beta}}\bar{W}_{\dot{\alpha}\dot{\beta}}S_{\gamma}^i \\
&\quad + \frac{i}{4}\varepsilon_{\alpha\beta}\bar{\nabla}_{\dot{\alpha}}^i\nabla^{\gamma}_{\dot{\gamma}}\bar{W}^{\dot{\gamma}\dot{\beta}}K_{\gamma\dot{\beta}} ,
\end{aligned} \tag{A.4d}$$

$$\begin{aligned}
[\nabla_{\alpha\dot{\alpha}}, \bar{\nabla}_{\dot{i}}^{\dot{\beta}}] &= i\delta_{\dot{\alpha}}^{\dot{\beta}}W_{\alpha\beta}\nabla_{\dot{i}}^{\beta} + \frac{i}{2}\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla_{\dot{i}}^{\beta}W_{\alpha\beta}\mathbb{D} - \frac{i}{4}\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla_{\dot{i}}^{\beta}W_{\alpha\beta}Y + i\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla^{\beta j}W_{\alpha\beta}J_{ij} \\
&\quad + i\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla_{\dot{i}}^{\beta}W^{\gamma}_{\alpha}M_{\beta\gamma} + \frac{i}{4}\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla_{\alpha i}\nabla^{\beta j}W_{\beta}^{\gamma}S_{\gamma j} - \frac{1}{2}\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla^{\beta}_{\dot{\gamma}}W_{\alpha\beta}\bar{S}_{\dot{i}}^{\dot{\gamma}} \\
&\quad + \frac{i}{4}\delta_{\dot{\alpha}}^{\dot{\beta}}\nabla_{\alpha i}\nabla^{\gamma}_{\dot{\gamma}}W_{\beta\gamma}K^{\beta\dot{\gamma}} .
\end{aligned} \tag{A.4e}$$

The complex superfield $W_{\alpha\beta} = W_{\beta\alpha}$ and its complex conjugate $\bar{W}_{\dot{\alpha}\dot{\beta}} := \overline{W_{\alpha\beta}}$ are superconformally primary, $K_A W_{\alpha\beta} = 0$, and obey the additional constraints

$$\bar{\nabla}_{\dot{i}}^{\dot{\alpha}}W_{\beta\gamma} = 0 , \quad \nabla_{\alpha\beta}W^{\alpha\beta} = \bar{\nabla}^{\dot{\alpha}\dot{\beta}}\bar{W}_{\dot{\alpha}\dot{\beta}} , \tag{A.5}$$

where

$$\nabla_{\alpha\beta} := \nabla_{(\alpha}^k \nabla_{\beta)k} , \quad \bar{\nabla}^{\dot{\alpha}\dot{\beta}} := \nabla_{\dot{k}}^{(\dot{\alpha}} \nabla^{\dot{\beta})k} . \tag{A.6}$$

When solving Bianchi identities the following list of non-vanishing torsion components are useful:

$$\begin{aligned}
T_{\alpha j}^{i\dot{\beta}a} &= -2i\delta_j^i(\sigma^a)_{\alpha}^{\dot{\beta}} , \\
T_{a\beta\dot{\gamma}}^{jk} &= -\frac{i}{2}\varepsilon^{jk}(\sigma_a)_{\beta}^{\dot{\beta}}\bar{W}_{\dot{\beta}\dot{\gamma}} , \quad T_{aj\dot{k}}^{\dot{\beta}\gamma} = -\frac{i}{2}\varepsilon_{jk}(\sigma_a)_{\beta}^{\dot{\beta}}W^{\beta\gamma} , \\
T_{ab\dot{k}}^{\gamma} &= \frac{1}{4}(\sigma_{ab})^{\alpha\beta}\nabla_k^{\gamma}W_{\alpha\beta} , \quad T_{ab\dot{\gamma}}^{\dot{k}} = \frac{1}{4}(\tilde{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}}\bar{\nabla}_{\dot{\gamma}}^{\dot{k}}\bar{W}_{\dot{\alpha}\dot{\beta}} .
\end{aligned} \tag{A.7}$$

B Component results from superspace

In this appendix, we summarize component results of the conformal superspace in [23]. The notation here differs from [23] and is summarized in [18] (see also [44]).

The supercovariant derivative is derived from the component projection of ∇_m

$$e_m{}^a\nabla_a| = \nabla'_m - \frac{1}{2}\psi_{m\dot{k}}^{\gamma}\nabla_{\gamma}^{\dot{k}}| - \frac{1}{2}\bar{\psi}_{m\dot{k}}^{\dot{\gamma}}\bar{\nabla}_{\dot{\gamma}}^k| + \mathfrak{f}_m{}^b K_b + \frac{1}{2}\phi_{m\gamma}^k S_k^{\gamma} + \frac{1}{2}\phi_{m\dot{k}}^{\dot{\gamma}} \bar{S}_{\dot{\gamma}}^{\dot{k}} , \tag{B.1}$$

where

$$\nabla'_m := \left(\partial_m + \frac{1}{2}\omega_m{}^{ab}M_{ab} + \phi_m{}^{ij}J_{ij} + iA_m Y + b_m \mathbb{D} \right) . \tag{B.2}$$

So that the projection of the covariant derivatives with Lorentz indices may be written as

$$\nabla_a| = \nabla'_a - \frac{1}{2}\psi_{a\dot{k}}^\gamma \nabla_\gamma^k| - \frac{1}{2}\bar{\psi}_{a\dot{k}}^{\dot{\gamma}} \bar{\nabla}_{\dot{\gamma}}^k| + \mathfrak{f}_a{}^b K_b + \frac{1}{2}\phi_{a\gamma}^k S_k^\gamma + \frac{1}{2}\phi_{a\dot{k}}^{\dot{\gamma}} \bar{S}_{\dot{\gamma}}^k, \quad (\text{B.3})$$

where

$$\nabla'_a := e_a{}^m \nabla'_m, \quad \bar{\psi}_{a\dot{k}}^{\dot{\gamma}} := e_a{}^m \bar{\psi}_{m\dot{k}}^{\dot{\gamma}}, \quad \mathfrak{f}_a{}^b := e_a{}^m \mathfrak{f}_m{}^a, \quad \phi_{a\alpha}^i := e_a{}^m \phi_{m\alpha}^i. \quad (\text{B.4})$$

Constraints on the form of the superspace torsion and curvatures imply that the connections $\omega_m{}^{ab}$, $\mathfrak{f}_m{}^a$ and $\phi_{m\alpha}^i$ are composite fields built out of the other components of the Weyl multiplet (see the torsion analysis of [23]). The spin connection is given by

$$\omega_{abc} = \frac{1}{2}(\mathcal{C}_{bca} + \mathcal{C}_{acb} - \mathcal{C}_{abc}) - \frac{1}{2}(\mathcal{T}_{bca} + \mathcal{T}_{acb} - \mathcal{T}_{abc}) - b_m e_a{}^m \eta_{ca} - b_m e_c{}^m \eta_{ba}, \quad (\text{B.5})$$

where

$$\mathcal{C}_{ab}{}^c := 2e_{[a}{}^n \partial_n e_{b]}{}^m e_{m\dot{c}}{}^c, \quad \mathcal{T}_{ab}{}^c := i(\sigma^c)_\gamma{}^{\dot{\delta}} \psi_{[a\dot{k}}^\gamma \bar{\psi}_{b]\dot{\delta}}^k. \quad (\text{B.6})$$

The S -supersymmetry connection is

$$\begin{aligned} \phi_{\beta\dot{\beta}\alpha}{}^j &= \frac{i}{12} \bar{\Psi}_\beta{}^{\dot{\gamma}}{}_\alpha{}^{\dot{\gamma}}{}^j + \frac{i}{6} \bar{\Psi}_\alpha{}^{\dot{\gamma}}{}_\beta{}^{\dot{\gamma}}{}^j + \frac{i}{12} \varepsilon_{\beta\alpha} \bar{\Psi}_\beta{}^{\dot{\gamma}}{}_\gamma{}^{\dot{\gamma}}{}^j \\ &\quad - \frac{1}{6} \bar{W}_{\dot{\beta}}{}^{\dot{\gamma}}{}_\psi{}_{\beta\dot{\gamma}}{}^j - \frac{1}{3} \bar{W}_{\dot{\beta}}{}^{\dot{\gamma}}{}_\psi{}_{\alpha\dot{\gamma}}{}^j + \frac{i}{2} \varepsilon_{\beta\alpha} \bar{\Sigma}_{\dot{\beta}}^j, \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} \bar{\phi}_{\beta\dot{\beta}\dot{\alpha}j} &= -\frac{i}{12} \Psi_{\dot{\beta}}{}^{\gamma}{}_{\gamma\dot{\alpha}}{}^{\beta}j - \frac{i}{6} \Psi_{\dot{\alpha}}{}^{\gamma}{}_{\gamma\dot{\beta}}{}^{\beta}j - \frac{i}{12} \varepsilon_{\dot{\beta}\dot{\alpha}} \Psi_{\beta}{}^{\dot{\gamma}}{}_\gamma{}^{\dot{\gamma}}{}^j \\ &\quad + \frac{1}{6} W_{\beta}{}^{\gamma}{}_\psi{}_{\gamma\dot{\beta}\dot{\alpha}}j + \frac{1}{3} W_{\beta}{}^{\gamma}{}_\psi{}_{\gamma\dot{\alpha}\dot{\beta}}j + \frac{i}{2} \varepsilon_{\dot{\beta}\dot{\alpha}} \Sigma_{\beta j}, \end{aligned} \quad (\text{B.8})$$

where we define the gravitino field strength

$$\Psi_{abk}{}^\gamma := e_a{}^m e_b{}^n \Psi_{mnk}{}^\gamma, \quad \Psi_{mnk}{}^\gamma := 2\nabla'_{[m} \psi_{n]k}{}^\gamma. \quad (\text{B.9})$$

The trace of the special conformal connection $\mathfrak{f}_m{}^a$ is (only the trace is required for our calculations):

$$\begin{aligned} \mathfrak{f}_a{}^a &= -D - \frac{1}{12} \hat{\mathcal{R}} + \frac{1}{24} \varepsilon^{mnpq} (\bar{\psi}_m{}^j \tilde{\sigma}_n \nabla'_p \psi_{qj}) - \frac{1}{24} \varepsilon^{mnpq} (\psi_{mj} \sigma_n \nabla'_p \bar{\psi}_q{}^j) \\ &\quad + \frac{i}{8} (\psi_{aj} \sigma^a \bar{\Sigma}^j) - \frac{i}{8} (\bar{\psi}_a{}^j \tilde{\sigma}^a \Sigma_j) + \frac{1}{12} W^{ab+} (\bar{\psi}_a{}^j \psi_{bj}) - \frac{1}{12} W^{ab-} (\psi_{aj} \bar{\psi}_b{}^j). \end{aligned} \quad (\text{B.10})$$

Here $\hat{\mathcal{R}} = \hat{\mathcal{R}}(e, \omega)$ corresponds to the Poincaré version of the Lorentz curvature

$$\hat{\mathcal{R}}_{ab}{}^{cd} := e_a{}^m e_b{}^n (\partial_{[m} \omega_{n]}{}^{cd} + \omega_{[m}{}^{cf} \omega_{n]f}{}^d). \quad (\text{B.11})$$

On a final note the following identity will prove useful:

$$\nabla_{(\alpha}^k W_{\beta\gamma)} = -2\Psi_{(\alpha\beta\gamma)}^k - i\Psi_{(\alpha\dot{\alpha}}{}^{\dot{\alpha}k} W_{\beta\gamma)}. \quad (\text{B.12})$$

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